The r-mode instability of the neutron star matter under the influence of URCA reactions

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Introduction

The r-mode oscillation in pulsar Neutron stars (NSs) is theoretically proposed to be continuous source of Gravitational wave (GW) emitted by these pulsars. There are different types of instabilities which can operate in a NS. Among them, the so-called r-mode instability, a toroidal mode of oscillation whose restoring force is the Coriolis force. [1,2]. The resulting instability in the pulsar star due to continuous emission of GW is counteracted due to the viscous effects in the NS. The r-mode oscillation evolves with a time dependence of \( e^{ωτ} \), where \( ω \) is the real part of the r-mode given by \( ω = \left( \frac{1}{τ} - 1 \right) (\frac{1}{τG} + \frac{1}{τSV}) \), \( τG \) is the gravitational time-scale and \( \frac{1}{τSV} \) is the sum of the reciprocal of all the different viscous time-scales. The viscous effect considered in this work are bulk and shear viscosities and the viscous effect coming from the crust-core boundary layer, \( \frac{1}{τVis} = \frac{1}{τBV} + \frac{1}{τSV} + \frac{1}{τGE} \), where \( τBV \) and \( τSV \) are bulk and shear time-scales of the core whereas \( τGE \) is the shear viscous time-scales of the crust-core boundary layer. It is to be noted here that out of all r-modes \( i=m=2 \) r-mode is important. The standard analytical expressions for \( τG, τBV, τSV \) and \( τGE \) are cited in ref [3]. From those expressions it can be seen that \( \frac{1}{τ} \) is a function of r-mode angular frequency \( ω \) and temperature \( T \).

The r-mode is stable for \( \frac{1}{τ} > 0 \) and it is unstable for \( \frac{1}{τ} < 0 \). So the instability boundary i.e. the critical frequency (\( ω_c \)) as a function of \( T \), can be obtained from the condition \( \frac{1}{τ} = 0 \). This has been shown in reference [3] for an approximate expression of the bulk viscosity \( ζ \) for different equation of state (EOS) constructed from the finite range simple effective interaction (SEI) for different value of slope parameter \( L(p_0) \).

In this work the instability boundary is calculated taking the contribution of URCA reaction into bulk viscosity \( ζ \) of the non-superfluid (npe\( μ \)) matter and is compared with the \( ω_c \) obtained for an approximate expression of \( ζ \) with the EOS of SEI.

Formalism

The URCA reactions are of two types, direct URCA (DURCA) and modified URCA (MURCA) reactions. DURCA is a sequence of two reactions,

\[ n \rightarrow p + l + \bar{ν}_l \] \( \rightarrow p + l \rightarrow n + \bar{ν}_l \quad \ldots (1) \]

where, lepton \( l \) is either \( e \) or \( μ \) and \( ν_l \) is the associated neutrino. DURCA is subject to the condition \( p_0 \leq p_n + p_p \), where \( p_n \) and \( p_p \) are the momentum of the neutron and proton, respectively and \( p_l \) is the momentum of the lepton, either \( e \) or \( μ \). In MURCA processes an additional nucleon required to conserve momentum, \( n + N \rightarrow p + N + l + \bar{ν}_l \) \( \rightarrow p + N + l \rightarrow n + N + \bar{ν}_l \), \( \ldots (2) \)

where, \( N \) is an additional nucleon required to conserve momentum of the reacting particles. MURCA is subject to the condition \( p_0 \leq 3p_n + p_p \). The total bulk viscosity is written as a sum of the
Results and Discussion

The instability boundary i.e. critical frequency ($\nu_c$) as a function of T can be calculated obtained from the condition $\sum \xi = 0$ using equation 3 for taking the contribution of URCA reaction into $\xi$ and is plotted for 1.4M⊙ and 1.8M⊙NS in panel (a) and (b) of Figure 1 respectively. Then it is compared with the $\nu_c$ which is calculated taking equation (4) into account for 1.4M⊙ and 1.8M⊙ NS. NSs shown in figure 1 are taken from Ref. [9]. It is found that the critical frequency decreases for temperature lower than 2.7 x 10^10 K for 1.4 M⊙ and 1.8 x 10^10 K for 1.8 M⊙ and increases for temperature higher than 2.7 x 10^10 K for 1.4 M⊙ and 1.8 x 10^10 K for 1.8 M⊙ for the bulk viscosity calculated from URCA reactions compared to the critical frequency calculated for general approximation of bulk viscosity. In the latter case all the NSs [9] are found to be in stable region while 4U 1608-522 is in unstable region for $\nu_c$ calculated with URCA reactions. It is also found that $\nu_c$ decreases in temperature lower than 2.7 x 10^10 K for 1.4 M⊙ and 1.8 x 10^10 K for 1.8 M⊙ and increases for temperature higher than 2.7 x 10^10 K for 1.4 M⊙ and 1.8 x 10^10 K for 1.8 M⊙ with increase in mass of the NS which can be seen in the figure 1.

Figure 1: The $r$-mode instability region for 1.4 and 1.8 M⊙ NS is shown for L = 76.26 MeV with the EOS of SEI.

Conclusion

A large increase in the $\xi$ takes place as the URCA process become operative and it becomes the dominating damping mechanism to the perturbation in NSs. As a result of which the critical frequency decreases in low temperature (T < 10^10 K) and increases in higher temperature (T >> 10^10 K) with comparison to the critical frequency calculated from approximate expression of $\xi$. With increases in mass of the NS the critical frequency is lowered for low temperature and increases in high temperature.

References