Model independent approach to inelastic photo pion production on deuteron

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Introduction

Measurements of tensor analysing powers T_0^2, T_1^2 and T_2^2 for inelastic photo pion reactions on deuteron have been measured [1-5] in recent years. The photo production of pions from deuterons was studied theoretically under Impulse Approximation using a phenomenological approach [6] as early as 1951. The problem was also studied [7] using the well known CGLN photo production amplitudes [8]. A statistical modelling [9] was used currently [1] to analyse some of the recent measurements. The reaction amplitude was calculated using the model described in [10].

The purpose of the present contribution is to outline a Model Independent theoretical approach to inelastic photo pion production on deuteron using irreducible tensor techniques [11].

Theoretical formalism

Let $\mathbf{k}=k\hat{\mathbf{k}}$ denotes the photon momentum which is chosen along the z-axis. The polarization of photon is denoted by $\mu = \pm 1$ following Rose [12]. Let $\mathbf{p_1}=p_1\hat{\mathbf{p}}_1$ and $\mathbf{p_2}=p_2\hat{\mathbf{p}}_2$ denote the c.m. momenta of the two nucleons and $\mathbf{Q}=Q\hat{\mathbf{Q}}$ the pion momentum in the final state. We may conveniently choose a right-handed cartesian coordinate system with $\mathbf{Q}$ coming out with an angle θ in the zx-plane. We choose Jacobi coordinates $\frac{1}{2}(\mathbf{p_1}+\mathbf{p_2}) = -\frac{1}{2}\mathbf{Q}$ and $\mathbf{p_f} = \frac{1}{2}(\mathbf{p_1}-\mathbf{p_2})$ to describe the three-body system in the final state. We consider $d\gamma \rightarrow pn\pi^0, pp\pi^-$ and $nn\pi^+$. The isospin state of the two nucleons in the final state is clearly I=1 in the last two cases whereas I could be 0 or 1 in the first. The reaction matrix for the inelastic photo pion production may then be written for each of I in the form

$$M(\mu) = \sum_{s=0}^{1} \sum_{\lambda=(1-s)}^{(1+s)} \left(S^\lambda(s,1)\cdot F^{s\lambda}(\mu)\right) \quad (1)$$

where the irreducible tensor operators defined following [11]. It is important to note that the each of these reactions can described by eight irreducible tensor amplitudes $F_\nu^{01}(\mu)$ and $F_\nu^{1\lambda}(\mu), \lambda = 0,1,2$ with the appropriate I and $\mu = \pm 1$ at all energies. The irreducible tensor amplitudes are expressible in terms of 2^L multipole partial wave amplitudes $F^j_{l(l_f s)j_f;L1}$ as

$$F_\nu^{Is\lambda}(\mu) = \sum_\alpha g_\alpha\, F^j_{l(l_f s)j_f;L1} \left(\left(\left(Y_{l_f}(\hat{\mathbf{p}}_f)\otimes Y_l(\hat{\mathbf{Q}})\right)^{L_f}\otimes Y_L(\hat{\mathbf{k}})\right)^\lambda_\nu \quad (2)$$

where α collectively denote $\{l, l_f, j_f, L_f, L, j\}$. The geometrical factor

$$g_\alpha = (4\pi)^3 i^{L-l-l_f}(-1)^{l+l_f+L-j+1}[j]^2[j_f][L_f][s]^{-1}\cdot W(Ls_iL_fs;j\lambda)W(sl_fjl;j_fL_f) \quad (3)$$

The partial wave amplitudes may be written as

$$F^j_{l(l_f s)j_f;L1} = \frac{1}{2}\left(P_+ M^j_{l(l_f s)j_f;L1} + i\mu P_- E^j_{l(l_f s)j_f;L1}\right) \quad (5)$$

and

$$P_\pm = \frac{1}{2}\{1 \pm (-1)^{L-l}\} \quad (6)$$

in terms of the magnetic and electric 2^L multipole amplitudes. At energies close to threshold, L=0 or 1. Pauli exclusion principle demands that $(-1)^{l_f+s+I}$ must be odd.

The initial deuteron polarization can be described by the spin density matrix

$$\rho^d = \frac{1}{3}\sum_{k=0}^{2}\left(S^K(1,1)\cdot t^K\right) \quad (7)$$

in terms of the Fano statistical tensors t_q^K. The differential cross-section for the reactions with each I and initially polarized deuteron is given by

$$\frac{d\sigma}{d\Omega} = \frac{1}{2}\sum_{\mu=\pm1} Tr\mathrm{M}(\mu)\rho^d\mathrm{M}^\dagger(\mu) \quad (8)$$

where

$$M^{\dagger}(\mu) = \sum_{s=0}^{1} \sum_{\lambda=(1-s)}^{(1+s)} (-1)^{s-1} \frac{[s]}{\sqrt{3}} \cdot \left(S^{\lambda}(1,s) \cdot (F^{\dagger})^{s\lambda}(\mu) \right) \quad (9)$$

and

$$(F^{\dagger})^{s\lambda}_{\nu} = (-1)^{\nu} F^{s\lambda *}_{-\nu} \quad (10)$$

Using standard Racah techniques, eq.(8) may be expressed in the form

$$\frac{d\sigma}{d\Omega} = \frac{\sqrt{3}}{6} \sum_{I,s,\lambda,\lambda',\mu,k=0}^{2} (-1)^{\Lambda}[\lambda][\lambda'] W(1ks\lambda; 1\lambda') \cdot \left(t^{K} \cdot \left(F^{s\lambda}(\mu) \otimes (F^{\dagger})^{s\lambda'}(\mu) \right)^{K} \right) \quad (11)$$

The differential cross section for unpolarized deuteron is readily identified with K=0 term given by

$$\frac{d\sigma_0}{d\Omega} = \frac{1}{6} \sum_{I,s,\lambda,\nu,\mu} \left| F^{s\lambda}(\mu) \right|^2 \quad (12)$$

We may now write $\frac{d\sigma}{d\Omega}$ in the form

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma_0}{d\Omega} \left[1 + \sum_{k=1}^{2} (t^{K} \cdot T^{K}) \right] \quad (13)$$

which define the analyzing powers T^{K}_{q} as

$$\frac{d\sigma_0}{d\Omega} T^{K}_{q} = \sqrt{3} \sum_{I,s,\lambda,\lambda',\mu} (-1)^{\lambda}[\lambda][\lambda'] W[1ks\lambda; 1\lambda'] \cdot \left(F^{s\lambda}(\mu) \otimes (F^{\dagger})^{s\lambda'}(\mu) \right)^{K}_{q} \quad (14)$$

The tensor analyzing powers T^2_0, T^2_1 and T^2_2 are given explicitly in terms of tensor amplitudes $F^{s\lambda}_{\nu}(\mu)$ through

$$\begin{aligned}\frac{d\sigma_0}{d\Omega} T^{2}_{q} = \sum_{I,\mu} [&-\sqrt{3} \left(F^{01}(\mu) \otimes (F^{\dagger})^{01}(\mu) \right)^{2}_{q} \\ &+ \left(F^{10}(\mu) \otimes (F^{\dagger})^{12}(\mu) \right)^{2}_{q} \\ &+ \left(F^{12}(\mu) \otimes (F^{\dagger})^{10}(\mu) \right)^{2}_{q} \\ &+ \frac{\sqrt{3}}{2} \left(F^{11}(\mu) \otimes (F^{\dagger})^{11}(\mu) \right)^{2}_{q} \\ &- \frac{3}{2} \left(F^{11}(\mu) \otimes (F^{\dagger})^{12}(\mu) \right)^{2}_{q} \\ &- \frac{3}{2} \left(F^{12}(\mu) \otimes (F^{\dagger})^{11}(\mu) \right)^{2}_{q} \\ &- \frac{\sqrt{7}}{2} \left(F^{12}(\mu) \otimes (F^{\dagger})^{12}(\mu) \right)^{2}_{q}]\end{aligned} \quad (15)$$

which is used to interpret the data in our formalism. Details will be presented.

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