Leptonic decays of pseudoscalar and vector heavy-light quarkonia

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1. Introduction: During the past few years, there is a growing interest in the experimental and theoretical studies of heavy-light mesons [1]**.** Further studies on heavy-light mesons are also important for the determination of Cabibo-Kobayashi-Maskawa (CKM) mass matrix elements. In this work we have calculated the leptonic decay constants of pseudoscalar and vector heavy-light quarkonia in the framework of a 4X4 Bethe-Salpeter equation, as a test of the wave functions [2] of these mesons that were analytically derived by us as solutions of their mass spectral equations in an approximate harmonic oscillator basis[2-3].

2. Leptonic decays of heavy-light pseudoscalar quarkonia: The leptonic decays of pseudoscalar quarkonia proceed through the coupling of the quark-anti quark loop to the axial vector current, and can be expressed through the 4D quark-loop integral as,

$$f_P P_\mu = \sqrt{3}\int \frac{d^4q}{(2\pi)^4} Tr\left[\psi_P(P,q)\gamma_\mu\gamma_5\right], \qquad (1)$$

where $\psi_P(P,q)$ is the full 4D Bethe-Salpeter wave function for pseudoscalar quarkonia. Making use of the fact that the 4D volume integral can be expressed as, $d^4q = d^3\hat{q}Md\sigma$, and integrating over the longitudinal component, $Md\sigma$, and making use of the fact that under Covariant Instantaneous Ansatz (CIA), the 3D BS wave function, $\psi_P(\hat{q})$ can be expressed as, $\psi_P(\hat{q}) = \int Md\sigma\psi_P(P,q)$, which is given as

$$\psi_P(\hat{q}) = N_P\left[M - i\gamma.P + i\frac{M(\omega_1-\omega_2)}{m_1\omega_2+m_2\omega_1}\gamma.\hat{q} + \frac{(m_1+m_2)}{\omega_1\omega_2+m_1m_2-\hat{q}^2}\gamma.P\gamma.\hat{q}\right]\gamma_5\varphi_P(\hat{q}) , \qquad (2)$$

evaluating trace over the Dirac gamma matrices, we can express the decay constant, f_P as a 3D integral,

$$f_P = 4\sqrt{3}N_P\int\frac{d^3\hat{q}}{(2\pi)^3}\varphi_P(\hat{q}) . \qquad (3)$$

where $\varphi_P(\hat{q})$ are the 3D BS wave functions of pseudoscalar mesons obtained by analytic solutions [2-3] of their 3D Mass spectral equations derived explicitly from the 4D Bethe-Salpeter equations shown explicitly in [2-3]. And N_P are the 4D BS normalizers obtained by the current conservation condition. The values of these decay constants are given in Table I.

Table I: Leptonic decay constants, f_P of ground state (1S) and excited states (2S) and (3S) of heavy-lightpseudoscalar mesons (in GeV.) in present calculation (BSE) along with experimental data

	BSE	Experiment
$f_{\eta_c}(1S)$	0.3282	0.335±0,075
$f_{\eta_c}(2S)$	0.2363	
$f_{\eta_c}(3S)$	0.1959	
$f_{B_c}(1S)$	0.4635	
$f_{B_c}(2S)$	0.3611	
$f_{B_c}(3S)$	0.3110	
$f_{B_S}(1S)$	0.2589	
$f_{B_S}(2S)$	0.2087	
$f_{B_S}(3S)$	0.1789	
$f_B(1S)$	0.2192	
$f_B(2S)$	0.1779	
$f_B(3S)$	0.1517	
$f_{Ds}(1S)$	0.2033	0.2546±0.0059
$f_{Ds}(2S)$	0.1360	
$f_{D_s}(3S)$	0.1079	
$f_D(1S)$	0.1751	0.2067±0.0089
$f_D(2S)$	0.1145	
$f_D(3S)$	0.0889	

2. Leptonic decays of heavy-light vector quarkonia: Leptonic decay constant of vector mesons is expressed through the coupling of the quark-antiquark loop to the vector current, which can expressed as the quark-loop integral,

$$f_V M\epsilon_\mu = \sqrt{3}\int \frac{d^4q}{(2\pi)^4} Tr\left[\psi_V(P,q)\gamma_\mu\right], \qquad (4)$$

where ϵ_μ is the polarization vector of the vector meson, satisfying the orthongonality condition, $\epsilon.P = 0$, with P being the momentum of the vector meson. Putting the 4D wave function $\psi_V(P,q)$, that involves the full Dirac structure of V-meson, and using a similar procedure as in pseudoscalar meson case, we can express the decay constant, f_V for vector meson as,

$$f_V = 4\sqrt{3}N_V \int \frac{d^3\hat{q}}{(2\pi)^3}\varphi_V(\hat{q}) \qquad (5)$$

where the 4D BS normalizer, N_V can again be obtained from the current conservation condition, following a similar procedure as in the case of pseudoscalar quarkonia. The results of f_V are given in Table II.

3. Discussions: In our recent works [2,3], we were not only interested in studying the mass spectrum of hadrons, which no doubt is an important element to study dynamics of hadrons, but also the hadronic wave functions [2], that play an important role in the calculation of decay constants, form factors, structure functions etc. for $Q\bar{Q}$, and $Q\bar{q}$ hadrons.

Table I: Leptonic decay constants, f_V of ground state (1S) and excited states (2S),…, (3S) of heavy-light vector mesons(in GeV.) along with data.

	BSE	Experiment
$f_{B_c^*}(1S)$	0.5171	
$f_{B_c^*}(2S)$	0.4292	
$f_{B_c^*}(1D)$	0.5441	
$f_{B_{c*}}(3S)$	0.3927	
$f_{B_S^*}(1S)$	0.3097	
$f_{B_S^*}(2S)$	0.2585	
$f_{B_S^*}(1D)$	0.2945	
$f_{B_S^*}(3S)$	0.2292	
$f_{B*}(1S)$	0.2473	
$f_{B*}(2S)$	0.2109	
$f_{B*}(1D)$	0.2359	
$f_{B*}(3S)$	0.1877	
$f_{DS*}(1S)$	0.2698	0.227±0.013
$f_{DS*}(2S)$	0.2026	
$f_{D_{S*}}(1D)$	0.2035	
$f_{DS*}(3S)$	0.1441	
$f_{D*}(1S)$	0.2251	0.196±0.011
$f_{D*}(2S)$	0.168	
$f_{D*}(1D)$	0.2035	
$f_{D*}(3S)$	0.1441	
$f_{J/\psi(1S)}$	0.4599	0.411±0.007
$f_{\psi(2S)}$	0.3256	0.279±0.008
$f_{\psi(1D)}$	0.2687	0.210±0.008
$f_{\psi(3S)}$	0.2218	

In present work [2], we have explicitly shown that the algebraic forms of 3D hadronic BS wave functions [2], calculated analytically by us as solutions of mass spectral equations of $Q\bar{q}$ systems, can not only provide information about the long range non-perturbative physics, but also tell us the shortest distance to which they can penetrate to in a hadron, due to which, they are not only physically reasonable, but also build a bridge [2] between the long range non-perturbative physics, and the short range perturbative physics. In the present work, we have used these wave functions to calculate leptonic decay constants, not only as a test of these wave functions [2], but also the overall framework. Our results are in reasonable agreement with data and other models

References:

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