Eigensolution of the modified screened Cornell potential and mass spectroscopy of the charmonium

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Introduction

The solution of the Schrödinger equation with different potential plays an important role in the number of branches of the physics and chemistry, such as molecular chemistry, nuclear and particle physics etc. The investigation of quarkonium systems is widely studied by using the solution of the Schrödinger equation. The solution of the interaction potential such as the Cornell potential [1], Cornell potential and harmonic oscillator potential [2] and extended Cornell potential [3] are much more applicable in the particle physics. Various analytical techniques have been adopted by different researchers to provide exact and approximate solutions to non-relativistic and relativistic wave equations with some typical potentials. These methods are supersymmetric quantum mechanics (SUSYQM) [4], asymptotic iteration method (AIM) [5], Nikiforov-Uvarov (NU) method [6, 7] etc. The Cornell potential is given as [1],

$$V(r) = Ar - \frac{B}{r} \tag{1}$$

We proposed modified screened Cornell potential as

$$V(r) = Are^{-\alpha r} - \frac{Be^{-\alpha r}}{r}$$

$$V(r) = \left(Ar - \frac{B}{r} - \alpha Ar^2 + \alpha B\right) \tag{2}$$

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Solutions of the D-dimensional Schrödinger equation for MSCP

NU method [7] is applicable to solve second-order differential equations or Dirac equation or Klein-Gordon equation. Here we used the NU method to solve the Scrödinger equation. D-dimensional schrödinger equation for spherically symmetric potential expresses as [7]

$$\frac{d^2R_{n\ell}(r)}{dr^2} + \frac{2\mu}{\hbar^2}\left[E_{n\ell} - V(r)\right]R_{n\ell}(r) - \frac{1}{r^2}\left[\frac{(D-1)(D-3)}{4} + \ell(\ell+D-2)\right] R_{n\ell}(r) = 0 \tag{3}$$

$$\frac{d^2R_{n\ell}(r)}{dr^2} + \frac{2\mu}{\hbar^2}\left[E_{n\ell} - Ar + \frac{B}{r} + \alpha Ar^2 - \alpha B\right] \times R_{n\ell}(r) - \frac{1}{r^2}\times \left[\frac{(D-1)(D-3)}{4} + \ell(\ell+D-2)\right] R_{n\ell}(r) = 0$$

Introducing an appropriate coordinate transformation $g = 1/r$, above equation can be written as

$$\frac{d^2R_{n\ell}(g)}{dg^2} + \frac{2g}{g^2}\frac{dR_{n\ell}(g)}{dg} + \frac{1}{g^4}\frac{2\mu}{\hbar^2}\times \left(E_{n\ell} - \alpha B + \frac{\alpha A}{g^2} - \frac{A}{g} + Bg - g^2T_1\right)\times R_{n\ell}(g) = 0 \tag{4}$$

In order to solve equation (4), we make the approximation scheme on the terms $1/g$ and $1/g^2$ [9] Setting $z = (g - \xi)$ and expanding in a power series around $z = 0$ with $\xi = \frac{1}{r_0}$,

$$\frac{d^2 R_{n\ell}(g)}{dg^2} + \frac{2g}{g^2}\frac{dR_{n\ell}(g)}{dg} +$$

$$\frac{1}{g^4}\left(-a + bg + cg^2\right) R_{n\ell}(g) = 0 \quad (5)$$

where $a = -\frac{2\mu}{\hbar^2}(E_{n\ell} - \alpha B + 6\alpha A/\xi^2 - 3A/\xi)$, $b = \frac{2\mu}{\hbar^2}(-8\alpha A/\xi^3 + 3A/\xi^2 + B)$ and

$$c = -\left(\frac{2\mu\left(-\frac{3A}{\xi^4} + \frac{A}{\xi^3}\right)}{\hbar^2} + T_1\right) \quad (6)$$

.

$$E_{n\ell} = \alpha B + \frac{3A}{\xi}\left(1 - \frac{2\alpha}{\xi}\right) + \frac{\hbar^2}{2\mu}\times$$

$$\left(\frac{\frac{2\mu T_1}{\hbar^2}}{(2n+1) \pm \sqrt{1 + 4\left[\frac{2\mu\left(-\frac{3A}{\xi^4} + \frac{A}{\xi^3}\right)}{\hbar^2} + T_2\right]}}\right)^2 \quad (7)$$

Where

$$T_1 = \left(-\frac{8\alpha A}{\xi^3} + \frac{3A}{\xi^2} + B\right),$$

$$T_2 = \frac{(D-1)(D-3)}{4} + \ell(\ell + D - 2) \quad (8)$$

Discussion and results

The properties of charmonuim meson have been computed. The following relations used for determining charmonium masses in the D-dimensional space for $\hbar = 1$.

$$M_c = 2m_c + \alpha B + \frac{3A}{\xi}\left(1 - \frac{2\alpha}{\xi}\right) - m_c\times$$

$$\left(\frac{T_1}{(2n+1) \pm \sqrt{1 + 4\left[m_c\left(-\frac{3A}{\xi^4} + \frac{A}{\xi^3}\right) + \right]}}\right)^2 \quad (9)$$

TABLE I: The mass spectra of charmonium $c\bar{c}$ in GeV for $m_c = 1.209$, $\xi = 4GeV, B = 1$, $\alpha = 0.25$, $D = 3$;

State	A	Our work	[10]	[11]	[12]	Exp. [13]
1S	1.632	3.0960	3.0961	3.096	3.107	3.096
1P	1.360	3.5112	3.2558	3.516		3.511
1D	1.7270	3.7709	3.5047	3.779		3.770
2S	1.6320	3.6864	3.6869	3.688		3.686

m_c and $m_{\bar{c}}$ is mass of the charm quark and anti charm quark respectively. Calculated results are in good agreement with experimental data and other theoretical studies[10–12].

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