Mass spectra and Decay widths of dimesonic hadron molecules

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1. Introduction

In the past few years, tremendous progress has been achieved experimentally in the multiquark states, specially in the heavy quark sector. Many theoretical attempts have been made to resolve the structure of multi-quark states as di-hadronic molecules. We have investigated the molecular structures of tetraquark states composed of heavy-light mesons. In the molecular framework, the masses and digamma decay widths of the meson-antimeson states are computed using the Hellmann potential.

2. Theoretical Methodology

The di-hadronic molecular system is assumed to be composed of meson and anti-meson. The interaction between hadrons bound in the molecular structure can be described by a Hellmann potential which is given by,

\begin{equation}
V(r) = -\frac{\alpha_s}{r} + B \frac{e^{-cr}}{r} + V_0
\end{equation}

Where \(\alpha_s\) and \(B\) are the strength of coulomb and Yukawa potential, respectively. \(c\) is the screening parameter. The same form of potential has been used by A.K.Rai et al. for light sector in ref [1]. In present calculation, the input parameter are taken from ref. [2] are listed in Table I. The binding energy of the di-hadronic state is obtained by solving schrödinger equation using mathematica notebook of Range-Kutta method. The mass of the di-hadronic(meson-anti meson) system is calculated as,

\begin{equation}
M = m_1 + m_2 + BE
\end{equation}

Here, \(m_1\) and \(m_2\) are the masses of the constituent mesons and anti-mesons, \(BE\) represents the binding energy of the di-mesonic system. Further, we have also employed hyperfine interaction for \(VV\) state. To remove degeneracy the spin dependent hyperfine interactions of confined one gluon exchange potential (COGEP) is expressed respectively as, [3],

\begin{equation}
\langle V_{h_1-h_2}\rangle = \frac{\sigma\langle \tilde{J}_{h_1}J_{h_2}JM|\tilde{J}_{h_1}\tilde{J}_{h_2}\tilde{J}_{h_1}J_{h_2}JM \rangle}{(E_{h_1} + m_{h_1})(E_{h_2} + m_{h_2})}
\end{equation}

\begin{equation}
M_{h_1-h_2} = M + E_{hyp}
\end{equation}

The digamma decay of the di-hadronic states are calculated using the wave function at the origin and it is given by [1],

\begin{equation}
\Gamma_{\gamma\gamma} = \frac{\pi\alpha^2}{m_{h_1}m_{h_2}} |\psi(0)|^2
\end{equation}

Where, \(m_{h_1}\) and \(m_{h_2}\) are the masses of constituent mesons.

\begin{table}[ht]
\centering
\caption{Parameter fitted in present model.}
\begin{tabular}{|c|c|}
\hline
Model parameter & Value \\
\hline
\hline
\(B\) & 1.0 \\
\hline
\(c\) & 0.134 GeV \\
\hline
\(V_0\) & 0.01 GeV \\
\hline
\(m_{\eta^0}\) meson & 1860 MeV \\
\hline
\(m_{\eta^0}\) meson & 2012 MeV \\
\hline
\(m_{D_s}\) meson & 1960 MeV \\
\hline
\(m_{D^+}\) meson & 2112 MeV \\
\hline
\end{tabular}
\end{table}

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### 3. Result and discussion

Using the Hellmann potential, we have solved the Schrödinger equation to compute the binding energy, masses, and digamma decay widths for di-hadronic states. We have used different combinations of di-hadronic states like $PP$, $PV$ and $VV$ - states to calculate the possible molecular structure of heavy-light mesons. The digamma decay width is evaluated using the wave function at the origin. Our calculated masses and decay widths are tabulated in Tables II and III respectively. Zhi-Hui Wang et al have studied the strong decays of $X(3940)$ and $X(4160)$ [7]. According to them, $X(3940)$ is a possible $\eta_c(3S)$ state. Additionally, they have also suggested that $X(4160)$ is difficult to consider it as $\eta_c(4S)$ state [7]. According to present analysis, the state $X(3940)$ is a possible $D_s - D_{0s}$ state having $I^G(J^{PC}) = 0^{--}/0^{--}$ (see Table II). Present calculation also suggests that $X(4160)$ state is a possible $D_s^* - D_{0s}^*$ molecular tetraquark state. The possible $I^G(J^{PC})$ for $X(4160)$ state is $0^+(2^{++})$ and the computed digamma decay width is 4.2765 keV. However, the experimental identification of the $(J^{PC})$ values for $X(3940)$ and $X(4160)$ states are still an open question. The state $X(3915)$ having $I^G(J^{PC}) = 0^+(0/2^{++})$ is an possible candidate of $D_s - D_s$ molecular state. As it’s $\gamma \gamma$ decay has been observed but not identified experimentally, we have computed the $\Gamma_{\gamma \gamma}$ as 4.7300 keV. The molecular structure of exotic tetra quark state could be tested only by the future quantitative experimental observations and the radiative $J/\psi$ decay.

### Acknowledgments

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### References


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**TABLE II:** Masses of di-mesonic states and comparison of computed results with other available theoretical results (in MeV)

<table>
<thead>
<tr>
<th>State</th>
<th>$^{(1)}F_c$</th>
<th>Present</th>
<th>Others</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_s - D_s$</td>
<td>0(0)</td>
<td>3.017</td>
<td>4918.4 ± 19.4 [6]</td>
</tr>
<tr>
<td>$D_s - D_s$</td>
<td>0(1++)</td>
<td>3.927 [6]</td>
<td></td>
</tr>
<tr>
<td>$D_s - D_s$</td>
<td>0(1--)</td>
<td>4.079 [6]</td>
<td></td>
</tr>
<tr>
<td>$D^* - D_s$</td>
<td>0(2++)</td>
<td>4.266</td>
<td></td>
</tr>
<tr>
<td>$D^* - D_s$</td>
<td>0(2--)</td>
<td>4.208 [6]</td>
<td></td>
</tr>
<tr>
<td>$D^* - D_s$</td>
<td>0(3++)</td>
<td>4.199</td>
<td></td>
</tr>
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<td>$D^* - D_s$</td>
<td>0(3--)</td>
<td>4.225 [6]</td>
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</tr>
</tbody>
</table>

**TABLE III:** Digamma decay width of meson-anti-meson molecule (in keV)

<table>
<thead>
<tr>
<th>State</th>
<th>$^{(1)}F_c$</th>
<th>Present</th>
<th>$\Gamma_{\gamma \gamma}$ (in keV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_s - D_s$</td>
<td>0(0)</td>
<td>4.730</td>
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<td>$D_s - D_s$</td>
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<td>4.380</td>
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<td>4.0741</td>
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<tr>
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<td>4.0741</td>
<td></td>
</tr>
<tr>
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<td>0(4++)</td>
<td>4.5560</td>
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</tr>
<tr>
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<td>0(4--)</td>
<td>4.5560</td>
<td></td>
</tr>
<tr>
<td>$D^* - D_s$</td>
<td>0(5++)</td>
<td>4.4891</td>
<td></td>
</tr>
<tr>
<td>$D^* - D_s$</td>
<td>0(5--)</td>
<td>4.4891</td>
<td></td>
</tr>
<tr>
<td>$D^* - D_s$</td>
<td>0(6++)</td>
<td>4.4891</td>
<td></td>
</tr>
<tr>
<td>$D^* - D_s$</td>
<td>0(6--)</td>
<td>4.4891</td>
<td></td>
</tr>
<tr>
<td>$D_{s} - D_s$</td>
<td>0(0)</td>
<td>4.626</td>
<td></td>
</tr>
<tr>
<td>$D_{s} - D_s$</td>
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</tr>
<tr>
<td>$D_{s} - D_s$</td>
<td>0(2--)</td>
<td>4.2765</td>
<td></td>
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