Calculation of Splittings in the masses of Bottomonium

Vipin Naik N. S.[12], A. P. Monteiro[2],* Praveen P. D'souza[12], and K. B. Vijaya Kumar[2]
[1] *Department of Studies in Physics, Mangalore University, Mangalagangotri - 574199, INDIA and*
[2] *P.G. Department of Physics, St. Philomena College, Puttur, Darbe - 574202, INDIA*

The mass spectra of Bottomonium is obtained using relativistic harmonic model. The Hamiltonian has comprised of confinement potential for larger distances and a two body confined one gluon exchange potential (COGEP) for shorter distances.

Hyperfine and spin-orbit splittings

The experimental value of hyperfine mass splitting for the ground state is given by [1]

$$M(^3S_1) - M(^1S_0) = 62.3 \pm 3.2 \text{ MeV} \quad (1)$$

The hyperfine mass splitting calculated in our model is in good agreement with both experimental data[1] and lattice QCD result (60.3 ± 7.7) MeV[?]. The EFTs have predicted a hyperfine mass splitting of $41 \pm 11^{+9}_{-8}$ MeV[?] which is lower than the experimental value. The calculated splitting and the comparison with the results of various models is listed in table I. The calculations of hyperfine splitting for S-wave has been done by using the formula, $\Delta_{hf}M(nS) = M(n^3S_1) - M(n^1S_0)$; meanwhile spin-orbit splitting for P-wave states of Bottomonium are computed by: $\Delta M(nP) = M(n^3P_2) - M(n^3P_1)$ and $\Delta M(nP) = M(n^3P_1) - M(n^3P_0)$.

To account for the confinement of the quarks in the bottomonium system we can estimate a characteristic quantity of the L=1 multiplets is the ratio between the two fine structures;

$$R = \frac{\Delta M(1^3P_2 - 1^3P_1)}{\Delta M(1^3P_1 - 1^3P_0)} \quad (2)$$

*Electronic address: aprakashmonteiro@gmail.com

Our estimate of the ratio is $\frac{\Delta M(1^3P_2-1^3P_1)}{\Delta M(1^3P_1-1^3P_0)}$ =0.33, and that from the experiment is 0.58.

TABLE I: **The hyperfine splitting and spin-orbit splittings in bottomonium states.**

	Mass Splittings	Present	Expt.	GI [2]	SP [3]
Hyperfine splittings	$M(1^3S_1 - 1^1S_0)$	61.03	64	63	71
	$M(2^3S_1 - 2^1S_0)$	23.38	24	27	29
	$M(3^3S_1 - 3^1S_0)$	16.04		18	21
	$M(4^3S_1 - 4^1S_0)$	12.42		12	
	$M(5^3S_1 - 5^1S_0)$	10		9	
Fine structure splittings	$M(1^3P_2 - 1^3P_1)$	15.48	19.43	20	21
	$M(1^3P_1 - 1^3P_0)$	46.98	33.34	30	21
	$M(2^3P_2 - 2^3P_1)$	11.38	13.19	10	18
	$M(2^3P_1 - 2^3P_0)$	31.62	22.96	20	25
	$M(3^3P_2 - 3^3P_1)$	9.97			16
	$M(3^3P_1 - 3^3P_0)$	23.79			22
	$M(4^3P_2 - 4^3P_1)$	6.84			
	$M(4^3P_1 - 4^3P_0)$	18.23			

Spin-dependent splittings

The spin averaged masses are defined by,

$$M(n\bar{S}) = \frac{3M(n^3S_1) + M(n^1S_0)}{4} \quad (3)$$

$$M(n\bar{P}) = \frac{3M(n^1P_1) + 5M(n^3P_2) + 3M(n^3P_1) + M(n^3P_0)}{12} \quad (4)$$

with n=1,2,3,... the radial quantum numbers.

The spin averaged masses calculated in our model are listed in the table II. The corresponding spin averaged mass and spin dependent splittings are estimated and is given in the table III.

TABLE II: **The spin averaged mass**

States	$n = 1$	$n = 2$	$n = 3$	$n = 4$	$n = 5$
nS (present)	9445.5	10018.65	10387.99	10701.40	10988.44
$n\bar{S}$ (experiment)[1]	9444.72	10017.19	-	-	-
nP (present)	9905.07	10276.22	10584.81	10846.30	11097.66
$n\bar{P}$ (experiment)[1]	9899.73	10260.13	-	-	-

TABLE III: **The Spin-averaged and -dependent splittings**

	States	Present	Expt.
Spin-averaged splittings	$\Delta M(1^1P_1 - 1\bar{S})$	453.45	454
	$\Delta M(1^3P_0 - 1\bar{S})$	411.25	414
	$\Delta M(1^3P_1 - 1\bar{S})$	458.23	447
	$\Delta M(1^3P_2 - 1\bar{S})$	473.71	467
Spin-dependent splittings	$\Delta M(1P - 1S)$	459.57	455
	$\Delta M(2\bar{P} - 2\bar{S})$	275.57	243
	$\Delta M(2\bar{S} - 1\bar{S})$	573.15	573
	$\Delta M(2\bar{P} - 1\bar{P})$	257.57	361

Discussions:

The correctness of theoretical description of the fine splitting structure in bottomonium can be verified by considering the hyperfine splitting in P-state. It has been found that the measured masses of 1^1P_1 and 2^1P_1 states of bottomonium practically coincide with the masses of spin-averaged triplet P-state:

$$\langle M(n^3P_J)\rangle = \frac{5M(1^3P_2) + 3M(1^3P_1) + M(1^3P_0)}{9} \quad (5)$$

Practically this is the measure of centroid of the three spin-orbit split states, $^3P_0$, $^3P_1$ and $^3P_2$. The correctness is computed by considering $\Delta_{hf}(\langle M(1^3P_J)\rangle - M(n^1P_1))$, where, spin-averaged triplet P-state is estimated to be $\langle M(1^3P_J)\rangle = 9907.11MeV$ and $\langle M(2^3P_J)\rangle = 10278.57MeV$. This yields P-state hyperfine splitting; $\Delta_{hf}(\langle M(n^3P_J\rangle - M(1^1P_1)) = 8.16MeV$ while the experimental value is $\Delta_{hf}(\langle M(1^3P_J)\rangle - M(1^1P_1)) = 1.62MeV$ and $\Delta_{hf}(\langle M(2^3P_J)\rangle - M(2^1P_1)) = 9.42MeV$ while the experimental value is $\Delta_{hf}(\langle M(2^3P_J)\rangle - M(2^1P_1)) = 0.48MeV$. Though our estimations are large when compared with the experimental, relatively less when compared with the hyperfine splitting in S-state.

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