Causal aspects of effective QCD models

Jayanta Dey* and Sabyasachi Ghosh
Indian Institute of Technology Bhilai, GEC Campus, Sejbahar, Raipur 492015, Chhattisgarh, India

Chowdhury Aminul Islam
Department of theoretical Physics, Tata Institute of fundamental Research, Homi Bhabha Road, Mumbai 400005, India

Let us start with energy momentum tensor of relativistic fluid,

$$T^{\mu\nu} = T_0^{\mu\nu} + \pi^{\mu\nu}, \quad (1)$$

where ideal part is $T_0^{\mu\nu} = \epsilon u^\mu u^\nu - P\Delta^{\mu\nu}$ and shear dissipation part is [7]

$$\pi^{\mu\nu} = \eta \mathcal{U}_\eta^{\mu\nu} , \quad (2)$$

commonly known as Navier-Stokes equation. If the fluid is quark matter, having N_f flavor and N_c color degeneracy factors, then from kinetic theory framework, energy-momentum tensor can be define as (for zero quark chemical potential)

$$T^{\mu\nu} = 4N_f N_c \int \frac{d^3\boldsymbol{p}}{(2\pi)^3} \frac{p^\mu p^\nu}{E} f_Q , \quad (3)$$

where a small deviation δf_Q of distribution function f_Q from equilibrium distribution f_Q^0 will be assumed, *i.e.*

$$f_Q = f_Q^0 + \delta f_Q . \quad (4)$$

By relating macroscopic Eq. (2) to microscopic Eq. (3) and identifying proper expression of δf_Q through relaxation time approximation (RTA) of relativistic Boltzmann equation (RBE), we will get

$$\eta = \frac{4N_F N_c}{15T} \int \frac{d^3\boldsymbol{p}}{(2\pi)^3} \left(\frac{\boldsymbol{p}^2}{E}\right)^2 \tau_Q f_Q (1 - f_Q) \quad (5)$$

*Electronic address: `jayantad@iitbhilai.ac.in`

Now, Eq. 2 violates causality. To check it, one can derive diffusion speed v_T from dispersion relation, based on Eq. (2) [1],

$$v_T(k) = 2\frac{\eta}{\epsilon + P} k \quad (6)$$

which can exceed speed of light at $k \to \infty$ with finite values of energy density ϵ and pressure P.

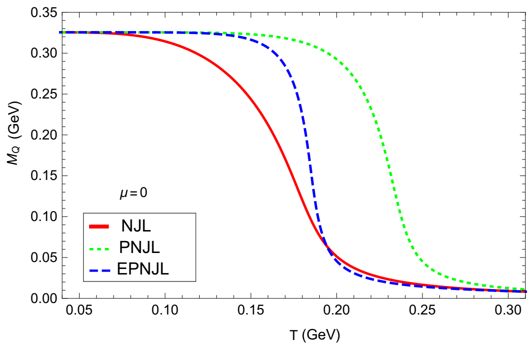

FIG. 1: Temperature dependence of constituent quark masses for NJL (solid line), PNJL (dotted line), EPNJL (dash line).

It was Maxwell [2] and Cattaneo [3], who replaced the algebraic Eq. (2) by time derivative equation, where causality can be maintained by introducing a finite values of shear relaxation time τ_π. In the limit of $\tau_\pi \to 0$, Maxwell-Cattaneo equation is merged to acausal Eq. (2). For this Maxwell-Cattaneo equation, one get diffusion speed at $k \to \infty$,

$$v_T^{max} \equiv \lim_{k\to\infty} \sqrt{\frac{\eta}{(\epsilon + P)\tau_\pi}} \quad (7)$$

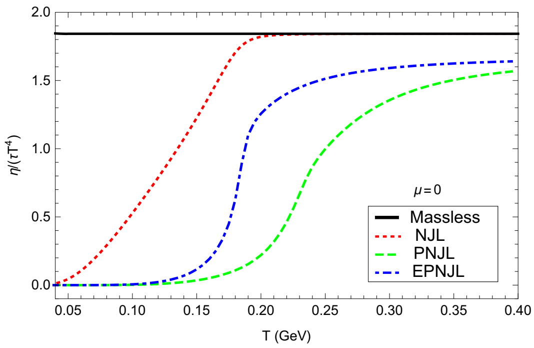

FIG. 2: Normalized shear viscosity $\eta/(\tau_c T^4)$ vs temperature T for massless quarks (black solid horizontal line), massive quarks, based on NJL (dotted line), EPNJL (dash-dotted line), PNJL (dash line).

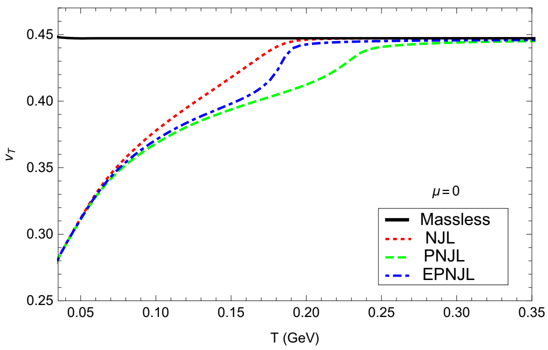

FIG. 3: Maximum value of causal diffusion speed v_T as a function of temperature for massless quarks (black solid horizontal line), massive quarks, based on NJL (dotted line), EPNJL (dash-dotted line), PNJL (dash line).

Interestingly, for all known fluids the limiting value of v_T^{max} has been found to be smaller than one. It means that diffusion speed of our known fluid never cross speed of light and causality is always preserved. In present investigation, we will explore on estimation of v_T^{max} for interacting quark matter, where interaction picture is mapped by different effective QCD models, like Nambu-Jona-Lasinio (NJL) model [4], Polyakov loop extended NJL (PNJL) model [5] and entangled PNJL (EPNJL) model [6]. These effective QCD models build a quasi-particle description of thermodynamics for interacting medium, where massless quark with Fermi-Dirac distribution is transformed to temperature dependent mass in NJL model and additional modified distribution functions in PNJL and EPNJL models. A detail discussion on these models are addressed in Ref. [7]. Using their constituent quark masses $M_Q(T)$, shown in Fig (1), in Eq. (5), we have estimated respective η's, which are displayed in Fig. (2). Assuming $\tau_\pi = \tau_Q$ in Eq. (7), we have plotted $v_T(T)$ for different model in Fig. (3). Which respect massless limit of $\eta \propto T^4$ and $v_T = 1/\sqrt{5}$, interacting values remain lower in low temperature domain. It reflects that massless non-interacting quark matter as well as interacting massive quark matter, based on different effective QCD models both obey causal description and investigation explore the zone with respect to the marginal values, beyond which causality can not be preserved.

References

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