Pair production from bremsstrahlung photon, and two-photon bremsstrahlung in relativistic ion-ion collisions

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Abstract: In this work, we are going to deal with electron-positron pair production from bremsstrahlung photon in high energy ion-ion collisions. We calculate the cross section for this process. This calculation is of relevance to the lepton- anti-lepton pair production process from bremsstrahlung photon in ion-ion collisions at RHIC, and proton-proton collisions at energies of Large Hadron Collider (LHC). We further study problem of two-photon bremsstrahlung emitted due to relativistic ion-ion collisions, and study the cross section, and the role of retardation and relativistic effects in the angular distributions of 2γ.

1. Pair-production from bremsstrahlung photon: Bremsstrahlung [1-4] is also known as breaking radiation, and is produced in ion-ion collisions. The energy emitted from the lighter ion is in the form of a bremsstrahlung photon [2], which further annihilates, and produces lepton-antilepton pair, if the energy of this photon is greater than two times the rest mass of the lepton. The purpose of our present work is to carry out the calculation of the differential cross section \( \frac{d^2\sigma}{d\Omega_{\ell\bar{\ell}}} \) and is relevant to ion-ion collisions studied at RHIC.

There are two Feynman diagrams contributing to this process in ion-ion scattering (shown in Fig.1). Here, Fig.1(a) corresponds to emission of bremsstrahlung photon, after the lighter ion interacts with the heavy ion, while the second diagram, Fig.1(b) corresponds to its emission, before the interaction. We can write the third order correction to the S-matrix for the ion-ion collision as,

\[
\begin{align*}
S_{fi} = \sqrt{\frac{m^2}{g_1 g_2 g_3 g_4}} M_{fi} (2\pi)^4 \delta^4(p_1 - p_3 - p_2 - p_4).
\end{align*}
\]

where we take the view, that when relativistically moving lighter ion scatters off the coulomb field of the heavy ion, the heavy suffers a very small recoil, owing to its heavier mass, which leads to a negligible Dirac current of heavy ion, that can be ignored. But this is sufficient to justify the four momentum conservation at the vertex, given by the four-momentum delta function. In Eq.(1), \( M_{fi} \) is the invariant amplitude, \( p_1 \) is the momentum of incoming lighter ion. When it interacts with heavy ion, it decelerates and produces three outgoing momenta, \( p_1', p_2, \) and \( p_2' \) where \( p_2 \) and \( p_2' \) are the final momenta of the lepton and anti-lepton respectively, that are produced from the bremsstrahlung photon. Thus, the invariant amplitude, \( M_{fi} \) for this process can be written as,

\[
\begin{align*}
M_{fi}^1 &= \bar{u}^{\gamma_1}(\vec{p}_1')(ie\gamma_\mu)(e\gamma_\nu A_\mu)\frac{(m_1^2 + m^2)}{(p_1' - q)^2 + m^2} u^{\gamma_2}(\vec{p}_2) \times \left[ -\frac{1}{q^2} \right] u^{\gamma_2}(\vec{p}_2')(i\gamma_\nu) v^{\gamma_2}(\vec{p}_2); \\
M_{fi}^2 &= \bar{u}^{\gamma_1}(\vec{p}_1')(i\gamma_\mu A_\mu)\frac{(m_1^2 + m^2)}{(p_1' - q)^2 + m^2} \times \left[ -\frac{1}{q^2} \right] (ie\gamma_\nu) u^{\gamma_2}(\vec{p}_2') v^{\gamma_2}(\vec{p}_2).
\end{align*}
\]
where $q = p_2 + p'_2$, is the momentum of the virtual photon. For kinematics of the process, we take the view that lepton, and anti-lepton are emitted back to back, and solving routine steps, we obtain the cross sectional formula,

$$\frac{d^2\sigma}{d\Omega_1 d\Omega_2} = \frac{2^6 \alpha_{em}^2 E_1}{\pi \epsilon_1^2 \cos^2 \frac{\theta_2}{2}} |\tilde{M}|^2 |p_1' = p_1 - p_2'|^2 \tag{3}$$

Here $|\tilde{M}|^2$ is the spin averaged invariant amplitude squared, which is expressed in terms of scalar products of various four momenta involved in the process, and is expressible in terms of the scattering angle $\theta_2$ between the incident ion momentum, $p_1$ and $p_2$, the momentum of the lepton created. While the angle between the incident ion momentum, $p_1$, and the created anti-lepton momentum, $p'_2$ is $\pi - \theta_2$. The cross section goes as third power of lepton momentum, $p_2$, and anti-lepton momentum, $p'_2$, is $|p_1' = p_1 - p_2'|^2$, after which it continues to decrease. When $Z$ is increased, the cross section is scaled by a factor $Z^2$. In Fig.2, we plot the graph of the differential cross section for production of electron-positron pair (through process shown in Fig.1), versus the scattering angle $\theta_2$, for incident ion energy, $E_1 = 2$ GeV, that scatters from the coulomb field of heavy ion of charge, $Z = 40$. For any given incident energy of electron, with the increase in scattering angle, the differential cross section increases, and peaks at angle, $\theta_2 = \frac{2\pi}{3}$, after which it continues to decrease. When $Z$ is increased, the cross section is scaled by a factor $Z^2$, though the overall features of the plot remain the same. However at higher energies, the charge form factors will play an increasingly important role. Detailed study of this effect will be relegated to a separate paper. Further from our plots of differential cross section versus the incident electron energy, $E_1$, it is observed that the differential cross section does not decrease and is nearly constant with increase in incident energy of the electron, $E_1$ from 0.0001 GeV, up to 10 GeV. for any scattering angle, $\theta_2$. This feature of our analysis is in complete conformity with the well known fact, that the main contribution to the differential cross section for bremsstrahlung and pair production processes is given by peripheral kinematics [2], whose contribution to differential cross section does not decrease when the energy of incident particle increases. Due to this feature, this work can also be employed to study processes with production of 2,...,6 particles by high energy $e^- e^+$ beams in colliders, that are widely used to study fundamental processes.

![Fig.1: The graph of differential cross section (in units of barns), and scattering angle, $\theta_2$ at $E_1 = 2$ GeV, and $Z = 40$.](image)

This problem is of relevance to the study of ion-ion collisions at energies of RHIC, where a lighter ion scatters off a heavier ion, and bremsstrahlung photon is emitted out from the lighter ion, which creates an ($e^- e^+$) or ($\mu^- \mu^+$) pair.

2. Two-photon bremsstrahlung process: We further calculated the four-fold differential cross section of the bremsstrahlung in relativistic ion collisions with respect to the incident photon energies, $\omega_{1,2}$ and the solid angles of the photons emission $\Omega_{1,2}$ given by

$$\frac{d^4\sigma}{d\omega_1 d\omega_2 d\Omega_1 d\Omega_2} = \frac{2^6 \epsilon_1^2 \epsilon_2^2 \alpha_{em}^2}{\pi^2 \epsilon_1 \epsilon_2 |\tilde{M}|^2} \int d\Omega_{p_1} |\tilde{M}_{f1}|^2$$

$$|\tilde{M}_{f1}|^2 = \frac{-e^6 |A_4|^2}{(|p_1 - k_2|^2 + m^2)^2 (|p_1 - k_1|^2 + m^2)^2} \times (4k_2, k_1 + k_2, p_1 - m^2 k_1, p'_1 + 1 - 2p'_1 k_2 - m^2)^2$$

and studied the role of retardation and relativistic effects in the angular distributions of two-photon bremsstrahlung [3], and our results are in broad agreement with [4].

References: