Study of quark star using quasiparticle model

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Introduction

We discuss the statistical mechanics and thermodynamics of quark matter at zero temperature and finite chemical potential using a thermodynamically consistent frame work of quasiparticle model for QGP without the need of any reformulation of statistical mechanics or thermodynamical consistency relation. to obtain the mass-radius relation of dense quarkstar. First we derive the quasiparticle equation of state at finite temperature [1] and finite chemical potential and then go to the $T = 0$ limit. This leads to a thermodynamically self consistent description of quark matter as an ideal fermi gas of quasiparticles. Then we apply the equation of state thus obtained to the quark star to obtain the mass-radius relation by solving Tolman-Oppenheimer-Volkoff (TOV) equation.

Quasiparticle model at T=0

As we are considering the case of zero temperature (T=0) and $m \neq 0$, the distribution functions become step functions and we can use standard integrals to express the results. Here we are considering the up, down, and strange quarks at zero temperature but at finite chemical potential, μ. So the particle density n and energy density ε at temperature $T = 0$ of a Fermi gas of free quasiparticles can be obtained as

$$n_i(\mu) = \frac{d_i}{6\pi^2}\,(\mu_i^2 - m_i^2)^{3/2} \qquad (1)$$

$$\varepsilon_i(\mu) = \frac{d_i}{2\pi^2}[\mu k_F(2\mu^2 - m_i^2) - m_i^4 \ln(\frac{k_F+\mu}{m_i})] \qquad (2)$$

where $\varepsilon = \sum_{i=u,d,s} \varepsilon_i$ with i denoting the quarks u,d,s with degeneracies $d_{u,d,s} = 2N_c = 6$. The Fermi momentum is $k_F = \sqrt{\mu^2 + m_i^2}$. Here we are considering the up, down and strange quarks with zero rest mass in the T=0 limit. Hence the effective mass becomes $m_i(\mu, T) \to m_i(\mu, T = 0) = m_i(\mu)$ which can be obtained from equation (5) as $m_i^2 = \frac{g^2\mu^2}{6\pi^2}$. This effective mass is obtained by the interaction of quarks with the other quarks of the dense system. g^2 is QCD coupling constant which is function of μ only.

The QCD scale parameter Λ_T in g for $n_f = 3$, has a value 200 MeV [2] . Note that the effective masses increase with the strong coupling constant g and the quark chemical potential μ. The pressure follows from the relation $\varepsilon = \mu\frac{\partial P}{\partial \mu} - P$ and we get

$$\frac{P}{\mu} = \frac{P_0}{\mu_0} + \int_{\mu_0}^{\mu} d\mu\,\frac{\varepsilon(\mu)}{\mu^2} \qquad (3)$$

where p_0 and μ_0 are the pressure and chemical potential at some reference point. As we have started from average number density and average energy density which are well defined quantities in statistical mechanics, there is no need of other extra quantities like bag constant to make the system thermodynamically consistent. With the aim to study the implication for the quark matter star, we consider a plasma of quarks and leptones and both charge neutrality and β equilibrium conditions have to be imposed. A process is said to be in β equilibrium if all process of beta decay of quarks involving electrons and antineutrinos

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and their inverse process are likely to occur at identical rate,

Though the strange quark is much heavier than the up and down quarks, all three quarks are approximated to be massless here. For massless u,d and s quarks

$$\mu_u = \mu_d = \mu_s, \mu_e = 0. \tag{4}$$

Also massless quark matter is charge neutral by itself.

1. Mass-radius relation of Quark stars

If the pressure in the inner region of neutron star could be high enough to cause a transition to a deconfined quark matter and if this were to happen through out the entire star, ie., not only in the innermost regions, a pure quark star would be the result. This is the kind of quark star that has been discussed here. If the density is high enough, quark matter is stable with respect to nuclear matter. When working with models of quark stars, temperature can generally be ignored, as the binding energy is many orders of magnitude larger than the thermal energy. To obtain the mass radius relation, we have to solve the TOV equation. The maximum mass of star can be found using this mass-radius diagram.

The standard equation describing the structure of quark star is Tolman-Oppenheimer-Volkoff equation. The equation is

$$\frac{dP}{dr} = \frac{[P(r) + \varepsilon(r)][M(r) + 4\pi r^3 P(r)]}{r[r - 2M(r)]} \tag{5}$$

$$M(r) = 4\pi \int_0^r \varepsilon(r) r^2 dr \tag{6}$$

where r is the radial coordinate, m is the mass up to the radius r of the star. P is the total pressure and ε is the energy density. The input for solving the mass-radius relation of the stars is the equation of state. This input can be obtained from the microscopic physics which gives the equation of state in the form $\varepsilon = \varepsilon(P)$, where P denotes the pressure and ε the energy density. The numerically calculated equation of state of the form $\varepsilon = \varepsilon(P)$

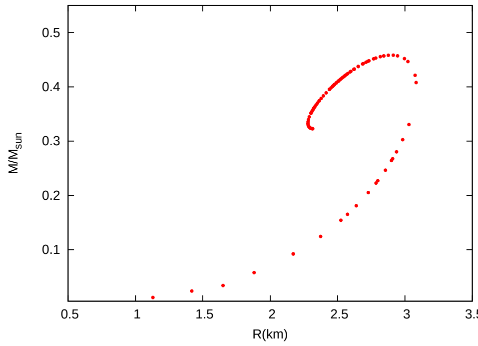

FIG. 1: The mass-radius relation of quark star

is obtained from equations (2) and (3). From this we can see how the microscopic physics affects the quantities measured in astrophysical experiments such as the mass-radius relation of the stars. The radius R of the star is the point where the pressure becomes zero, $P(r = R) = 0$ and M is the corresponding mass ie., M = m(R).

2. Results and conclusion

The parmetric plot for the mass-radius relation is shown in fig 5. It is found that the maximum mass obtained is about $0.46 M_{sun}$and corresponding radii is about 2.9 Km as obtained in literature [3]. Similar result was obtained in [4] also, but there they used the quasiparticle model with reformulated statistical mechanics.

References

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