NJL model estimation of anisotropic electrical conductivity for quark matter in presence of magnetic field

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Present work has gone through the microscopic calculation of electrical conductivity of quark matter in presence of magnetic field, where Nambu-Jona-Lasinio model is considered for mapping the interaction picture of the medium. Let us start with Ohm’s law

\[ J^i_D = \sigma^{ij} E_j \]  

where \( J^i_D \) is dissipative current, \( \sigma^{ij} \) electric conductivity tensor and \( E_j \) is electric field. Now for a fluid of quark having spin-color degeneracy \( g \) and electric charge \( q_f \) dissipative current from kinetic theory framework can be written as,

\[ J^i_D = q_f g \int \frac{d^3p}{(2\pi)^3} \vec{v} \delta f \]  

Where \( \delta f \) is small deviation of quark distribution function from the equilibrium Fermi-Dirac distribution of quark \( f_0 = \frac{1}{e^{\frac{\mu}{T}} + 1} \). In terms of 3-momentum (\( \vec{p} \)) and energy (\( \omega \)) particle velocity can be written as \( \vec{v} = \frac{\vec{p}}{\omega} \) with \( \omega = \sqrt{\vec{p}^2 + M^2} \). Now to find \( \delta f \) in presence of electric field \( \vec{E} \) and magnetic field \( \vec{B} \) we use relaxation time approximation (RTA) in Boltzmann’s equation, where we can assume a general force term

\[ \vec{F} = \alpha \vec{e} + \beta \vec{b} + \gamma \vec{e} \times \vec{b} \]  

where \( \vec{e}, \vec{b} \) are unit vectors along \( \vec{E} \) and \( \vec{B} \). Connecting \( \delta f \) and \( \vec{F} \) suitably [1, 2], the coefficients \( \alpha, \beta \) and \( \gamma \) can be found as

\[ \alpha = q \left( \frac{\tau_c}{\omega} \right) \frac{1}{1 + (\tau_c/\tau_B)^2} \vec{E}, \]

\[ \beta = q \left( \frac{\tau_c}{\omega} \right) \frac{(\tau_c/\tau_B)^2}{1 + (\tau_c/\tau_B)^2} (\vec{e} \cdot \vec{B}) \vec{E}, \]

\[ \gamma = -q \left( \frac{\tau_c}{\omega} \right) \frac{(\tau_c/\tau_B)^2}{1 + (\tau_c/\tau_B)^2} \vec{E}, \]  

where \( \tau_B \) and \( \tau_c \) as magnetic and thermal relaxation time. After taking care of all degeneracy factors of \( u \) and \( d \) quarks, we get the 3 components of electrical conductivity, whose general expressions can be written as

\[ \sigma_n = c^2 \beta \frac{20}{9} \int \frac{d^3p}{(2\pi)^3} \omega^2 \left( \frac{\tau_c}{\tau_B} \right)^n f_0(1-f_0) \]  

with \( n = 0, 1, 2 \). We will use temperature (\( T \)) and magnetic field (\( B \)) dependent effective quark mass from NJL model, briefly discussed below, in Eq. (5) to estimate \( \sigma_n \) of quark matter.

The Lagrangian density for the isospin-symmetric (\( m_u = m_d \)) two-flavor version of NJL model in presence of electromagnetic field \( (A^\mu) \) is given by

\[ L_{NJL} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \bar{\psi} (\gamma^5 \vec{D} - m) \psi + G \left( (\bar{\psi}\psi)^2 + (\bar{\psi}\gamma_5 \vec{T}\psi)^2 \right), \]  

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FIG. 1: Temperature dependence of three components of electrical conductivities \((\sigma_{0,1,2})\) with \(eB = 0.2\ \text{GeV}^2\) and \(\sigma\) (without \(B\)) for \(\tau_c = 5\ \text{fm}\) (b) and \(0.2\ \text{fm}\) (d). \(\Delta \sigma_n/\sigma = (\sigma_n - \sigma)/\sigma\) for \(\tau_c = 5\ \text{fm}\) (a) and \(0.2\ \text{fm}\) (c).

Where, \(D_\mu = i\partial_\mu - QA_\mu\) with \(Q = \text{diag}\(q_u = 2e/3, q_d = -e/3\)\) as the charge matrix. In quasi-particle approximation, the gap equation for the constituent quark mass \(M\) at finite \(T\) and \(B\) is given by

\[
M(B,T) = m - 2G(B,T) \sum_{f=u,d} \langle \bar{\psi}_f \psi_f \rangle, \quad (7)
\]

where \(\langle \bar{\psi}_f \psi_f \rangle\) represents the quark condensate of flavor \(f\), and a thermo-magnetic NJL coupling constant \(G(B,T)\) has been considered [3, 4]. In Figs. 1(b) and (d), we present the temperature dependence of the different components of electrical conductivities \(\sigma_n\) (scaled with \(T\)) in presence of an external magnetic field as well as the without field case,

\[
\sigma = e^2 \beta \frac{20}{9} \int \frac{d^3p}{(2\pi)^3} \frac{p^2}{|\vec{p}|} T \tau_c f_0(1 - f_0). \quad (8)
\]

The difference \(\Delta \sigma_n/\sigma = (\sigma_n - \sigma)/\sigma\) are also plotted in Figs. 1(a) and (c) which emphasize the effect of external magnetic field on each components. The results are presented with a fixed value of \(eB = 0.2\ \text{GeV}^2\) but for two different values of \(\tau_c\), i.e. \(\tau_c = 5\ \text{fm}\) (a,b) \(\tau_c = 0.2\ \text{fm}\) (c,d), which can be assigned with the zones \(\tau_c > \tau_B\) and \(\tau_c < \tau_B\) respectively. It means that \(eB = 0.2\ \text{GeV}^2\) may be considered as stronger magnetic field for \(\tau_c = 5\ \text{fm}\) and weaker magnetic field for \(\tau_c = 0.2\ \text{fm}\). Therefore, former case is showing \(\sigma_2 > \sigma_0\) and latter case is showing \(\sigma_2 < \sigma_0\). It is controlled by the anisotropic function \(eB/|\vec{p}|\). In terms of the anisotropy, the above outcomes can be briefly sketched, or:

- for \(\tau_c = 5\ \text{fm}\)
  \[
  \sigma^{xx} = \sigma^{yy} < \sigma^{zz} \Rightarrow \text{larger anisotropy} \quad (9)
  \]
- for \(\tau_c = 0.2\ \text{fm}\)
  \[
  \sigma^{xx} = \sigma^{yy} \approx \sigma^{zz} \Rightarrow \text{smaller anisotropy} \quad (10)
  \]

when external magnetic field \(eB = 0.2\ \text{GeV}^2\) is along the z-direction. As seen from Fig. 1, all the components of the electrical conductivities increase with temperature with a kink near the quark-hadron phase transition temperature \(T_c\) and rate of increments are also different for two different phases.

References


