Viscous coalescence model for relativistic heavy-ion collisions

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Introduction

Recombination models, along with fragmentation processes, have been used quite successfully to describe hadronization in heavy ion collisions. We have incorporate viscous corrections to the coalescence model for hadron production from a dissipative quark-gluon plasma. We use this viscous coalescence model to fit the spectra and elliptic flow of hadrons for 2.76 TeV Pb-Pb collisions at LHC.

Coalescence Model

Coalescence model in heavy ion collision is mainly based on an instantaneous projection of thermalised quark states, those are close to each other both in space and in momentum space, onto hadron states [1]. This model characterize numerous salient features of hadronisation in heavy-ion collisions, including baryon enhancement [2] and the robust scaling of the elliptic flow with the number of valence quarks. It has been argued that the flow anisotropy originates in the partonic phase and it obeys a simple valence quark scaling for low transverse momentum, that naturally arises from a recombination model [3]. However, in this model that densely populated phase space distribution of partons do not change with hadronization, there are no dynamical thermal gluons in the medium and QCD plays a background part. Under these assumptions, temperature merely plays any part than scale the momentum.

Formalism

In order to consider a boost invariant framework, it is easier to work in the Milne coordinate system where,
\[ \tau = \sqrt{t^2 - z^2}, \quad \eta_s = \tanh^{-1}(z/t), \]
\[ r = \sqrt{x^2 + y^2}, \quad \varphi = \text{atan2}(y, x). \]

The metric tensor for this co-ordinate system is
\[ g_{\mu\nu} = \text{diag}(1, -\tau^2, -1, -r^2). \]

Boost invariance and rotational invariance implies \( u^\varphi = u^{\eta_s} = 0 \). In summary, the hydrodynamic fields are parametrized as
\[ T = T_1, \quad u^\tau = \gamma_T \beta_T, \]
\[ u^\varphi = u^{\eta_s} = 0, \quad u^r = \gamma_T, \]
where \( R \) is the transverse radius of the fireball at freeze-out, \( \gamma_T = 1/\sqrt{1 - \beta_T^2} \) is the Lorentz factor in the transverse direction and \( \beta_T \) is the transverse expansion velocity. For non-central collisions, the transverse fluid velocity profile can be parametrized as
\[ \beta_T = \beta_0 \left( \frac{r}{R} \right)^m \left[ 1 + 2 \sum_{n=1}^{\infty} \beta_n \cos[n(\varphi - \psi_n)] \right], \]
where \( \beta_n \) are the strength of flow anisotropies in the transverse direction and \( \psi_n \) are the angles between the x axis and the major axis of the participant distribution. However, in the present calculation we can only consider elliptic flow and treat \( \beta_2 \) as a parameter. The distribution function with viscous correction is written as \( f = f_0 + \delta f \). Approximating the shear stress tensor with its first-order relativistic Navier-Stokes expression, \( \pi_{\alpha\beta} = \)
\[ 2\eta \nabla \langle u_{\alpha}u_{\beta} \rangle, \] the expression for the Grad’s 14-moment approximation reduces to [4, 5]

\[ \delta f^{(1)}_G = \frac{f_0 f_0}{T^2 (u \cdot p)} \left( \frac{y}{u} \right) p^\alpha p^\beta \nabla \langle u_{\alpha}u_{\beta} \rangle, \] (1)

whereas that due to the Chapman-Enskog method leads to [6]

\[ \delta f^{(1)}_{CE} = \frac{5 f_0 f_0}{T^2 (u \cdot p)} \left( \frac{y}{u} \right) p^\alpha p^\beta \nabla \langle u_{\alpha}u_{\beta} \rangle. \] (2)

For a particle at the space-time point \((\tau, \eta, r, \phi)\) with the four momentum \(p^\mu = (m_T \cosh y, p_T \cos \varphi_p, p_T \sin \varphi_p, m_T \sinh y)\), we get

\[ p_\tau = m_T \cosh (y - \eta), \]
\[ p_\eta = -m_T \sinh (y - \eta), \]
\[ p_r = -m_T \cos (\varphi_p - \varphi), \]
\[ p_\varphi = -m_T \sin (\varphi_p - \varphi), \]
\[ \nabla (\tau u^\tau) = \frac{(u^\tau)^2}{3} \left[ \frac{u^\tau}{\tau} + (1 - 2m) \frac{u^r}{r} \right], \]
\[ \nabla (\eta u^\eta) = \frac{1}{3 \tau^2} \left[ (m + 1) \frac{u^\tau}{\tau} - 2 \frac{u^r}{r} \right], \]
\[ \nabla (\varphi u^\varphi) = \frac{(u^\varphi)^2}{3} \left[ \frac{u^\tau}{\tau} + (1 - 2m) \frac{u^r}{r} \right], \]
\[ \nabla (\varphi u^\varphi) = \frac{1}{3 \tau^2} \left[ \frac{u^\tau}{\tau} + (m - 2) \frac{u^r}{r} \right], \]
\[ \nabla (\varphi u^\varphi) = \beta \tau \nabla (\varphi u^\varphi), \]
\[ \nabla (\varphi u^\varphi) = (u^\varphi)^3 \beta \frac{(\varphi)}{r^2} \left( \frac{r}{\tau} \right)^m \ldots \]
\[ \ldots \sum_{n=1}^{\infty} n \beta_n \sin [n(\varphi - \psi_n)], \]
\[ \nabla (\varphi u^\varphi) = \beta \tau \nabla (\varphi u^\varphi). \]

The freeze-out hyper-surface is \(d\Sigma_\mu = (\tau d\eta, r dr d\varphi, 0, 0, 0)\), and therefore the integration measure is given by

\[ p^\mu d\Sigma_\mu = m_T \cosh (y - \eta) \tau d\eta r dr d\varphi. \]

**Model with viscous correction**

Momentum distribution of number density is given by,

\[ \frac{d^2 N}{d^2 p_T dy} = \frac{1}{(2\pi)^3} \int_0^R r dr \int_0^{2\pi} d\phi \ldots \]
\[ \ldots \int_{-\infty}^{\infty} \tau d\eta, m_T \cosh (y - \eta) (f_0 + \delta f). \] (3)

For Coalescence Model distribution functions are modified as, \(f(u, E, \frac{p}{T}) \rightarrow\)

\[ \left\{ \begin{aligned}
&f(u, E, \frac{p}{T}) f(u, E, \frac{p}{T}) \quad \text{meson}, \\
&f(u, E, \frac{p}{T}) f(u, E, \frac{p}{T}) f(u, E, \frac{p}{T}) \quad \text{baryon}.
\end{aligned} \right. \]

The anisotropy flow is given by,

\[ v_n (p_T) = \frac{\int_{-\pi}^{\pi} d\phi \cos [n(\phi - \psi_n)] \frac{dN}{dp_T dp_T d\phi}}{\int_{-\pi}^{\pi} \frac{dN}{dp_T dp_T d\phi}}. \] (4)

\[ v_2 \text{ vs } p_T \text{ of pions at LHC for 20-30 % centrality (Data)}. \]

The fit parameters, we get from this analysis, are in reasonable agreement with literature.

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**References**