Susceptibility of quark matter at non-zero temperature

Kausik Pal*

Department of Physics, Serampore College, Serampore 712201, India.

I. Introduction

In recent years, in the area of high energy physics research, significant progress has been made to understand the properties of QCD matter leading to major advancement in the theoretical front addressing some of the subtle issues of the quasiparticles excitation in the extreme conditions of temperature and/or density. The experimental search of such matter needs reliable theoretical estimates of various signals which depend on the energy density, pressure, temperature etc. In the present work, we calculate the finite temperature corrections to the spin susceptibility including the correlation terms [1, 2]. This requires the knowledge of the ground state energy density at finite temperature of spin polarized matter with the inclusion of bubble diagrams [1], like what one does for the calculation of the correlation energy for degenerate electron gas [3], which serves as the starting point of the present article [1, 2, 4, 5].

II. Susceptibility with finite temperature

In this section we calculate the finite temperature corrections to the energy density and spin susceptibility including the correlation corrections term within the one-gluon-exchange model for non-zero chemical potential [1, 4, 5]. We assume the QCD matter is composed of light quarks only, *i.e.* the up and down quarks. Due to asymptotic freedom, one may expect that the quarks and gluons interact weakly. Thus, the properties of QCD might be computable perturbatively. So in our calculations it is assumed QCD coupling constant $\alpha_c = \frac{g^2}{4\pi} < 1$. It is to be mentioned, that for weak coupling constant g, perturbation theory can only be worked out to a finite order. In the strong coupling limit, the perturbation theory fails and one has to resort to lattice results. However, in the present work we assume the coupling to be weak enough for the series expansion in terms of α_c to converge. Although the perturbative expansion converge very slowly, the approach makes predictions consistent, when comparison is possible, with lattice results which do not contain such an approximation [6].

We consider the color symmetric forward scattering amplitude of two quarks around the Fermi surface by the one gluon exchange interaction. The direct term does not contribute as it involves the trace of single color matrices, which vanishes, while the leading contribution comes from the exchange and correlation term [2, 4]. Since our system is ultra-relativistic, in this limit, the angular averaged interaction parameter is given by [5]

$$f_{pp'}^{ss'} = \frac{g^2}{9pp'}\int\frac{\mathrm{d}\Omega_1}{4\pi}\int\frac{\mathrm{d}\Omega_2}{4\pi} \Big[1+(\hat{p}\cdot\hat{s})(\hat{p}'\cdot\hat{s}')\Big]. \quad (1)$$

The leading contributions to the energy density of quarks are given by three terms viz. kinetic, exchange and correlation energy densities, i.e.

$$E_q = E_{kin} + E_{ex} + E_{corr}. \quad (2)$$

The total kinetic energy density for spin-up and spin-down quarks, including the color and flavor degeneracy factors for quarks, is

$$E_{kin} = \frac{3}{(2\pi)^2}\Big\{p_f^4[(1+\xi)^{4/3}+(1-\xi)^{4/3}] + 2\pi^2T^2p_f^2[(1+\xi)^{2/3}+(1-\xi)^{2/3}]\Big\}. \quad (3)$$

Here, $\xi = (n_q^+ - n_q^-)/(n_q^+ + n_q^-)$ is the spin polarization parameter with $0 \le \xi \le 1$. n_q^+ and

*Electronic address: kausik.sinp@gmail.com

n_q^- correspond to the densities of spin-up and spin-down quarks, respectively and p_f is the Fermi momentum of the unpolarized matter ($\xi = 0$).

The analytical expression for the total exchange energy density is found to be

$$E_{ex} = \frac{g^2}{(2\pi)^4} \Big\{ p_f^4 [(1+\xi)^{4/3} + (1-\xi)^{4/3} + 2(1-\xi^2)^{2/3}] + \frac{4}{3}\pi^2 T^2 p_f^2 [(1+\xi)^{2/3} + (1-\xi)^{2/3}] \Big\}. \quad (4)$$

The spin susceptibility can be determined by the change in energy density of the system as quarks spin are polarized. In the small ξ limit, the energy density behaves like [2, 5]

$$E(\xi) \;=\; E(\xi = 0) + \frac{1}{2}\beta_s \xi^2 + \mathcal{O}(\xi^4), \quad (5)$$

where β_s is the spin stiffness constant. The spin susceptibility χ is inversely proportional to the spin stiffness; mathematically $\chi = 2\beta_s^{-1}$. The spin susceptibility can be written as [2, 5]

$$\chi^{-1} \;=\; \chi_{kin}^{-1} + \chi_{ex}^{-1} + \chi_{corr}^{-1}. \quad (6)$$

With the help of Eq.(5), each energy contribution to the susceptibility is

$$\begin{aligned} \chi_{kin}^{-1} &= \frac{p_f^4}{3\pi^2}\Big(1 - \frac{\pi^2 T^2}{p_f^2}\Big), \\ \chi_{ex}^{-1} &= -\frac{g^2 p_f^4}{18\pi^4}\Big(1 - \frac{\pi^2 T^2}{3p_f^2}\Big), \\ \chi_{corr}^{-1} &= -\frac{(g^4 \ln g^2) p_f^4}{576\pi^6}\Big(1 + \frac{4\pi^2 T^2}{3p_f^2}\Big). \quad (7) \end{aligned}$$

Using Eq.(6) and Eq.(7), the sum of all the contribution to the susceptibility is given by

$$\chi \;=\; \chi_P \Big[1 - \frac{g^2}{6\pi^2}\Big(1 + \frac{4\pi^2 T^2}{3p_f^2} + \frac{\pi^4 T^4}{3p_f^4}\Big) - \frac{g^4 \ln g^2}{192\pi^4}\Big(1 + \frac{7\pi^2 T^2}{3p_f^2} + \frac{4\pi^4 T^4}{3p_f^4}\Big)\Big]^{-1} \quad (8)$$

where χ_P is the Pauli susceptibility [2].

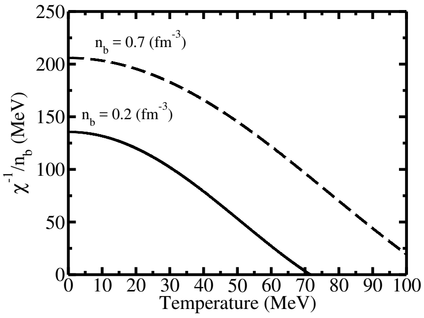

FIG. 1: The temperature dependence of inverse spin susceptibility.

III. Results and Discussions

The numerical estimation of χ^{-1} per baryon is given in Fig. 1 for two different densities, 0.2 fm^{-3} and 0.7 fm^{-3}, respectively. In figure $n_b = \frac{1}{3}n_q$, is the baryon density. We see susceptibilities blow up at different temperatures for two different densities. This can be identified as a physical instability of the QCD matter towards an ordered phase.

Acknowledgments
I am indebted to late A.K.Dutt-Mazumder who introduced me to this topic.

References

[1] K. Pal, S. Biswas and A.K. Dutt-Mazumder, Phys.Rev.C **80**, 024903 (2009).
[2] K.Pal, Eur.Phys.J.Plus **130**, 23 (2015).
[3] M. Gell-Mann and K.A. Brueckner, Phys. Rev. **106**, 364 (1957).
[4] K.Pal, S.Biswas and A.K.Dutt-Mazumder, Phys.Rev.C **79**, 015205 (2009).
[5] K. Pal and A.K. Dutt-Mazumder, Phys. Rev. C **80**, 054911 (2009).
[6] F. Karsch, Nucl. Phys. A **698**, 199 (2002).