Coherence measure of neutrino oscillations

S.Joshi* and S.R.Jain
Nuclear Physics Division, Bhabha Atomic Research Centre, Mumbai - 400085, INDIA and
Homi Bhabha National Institute, Anushakti Nagar, Mumbai 400094, INDIA

Neutrino oscillations provide a wonderful tool to study the intricacies of quantum mechanics at macroscopic distance scales. One such phenomena at the heart of quantum mechanics is that of interference. In its more modern form interference has been related to a measure of coherence [1, 2].

The quantum information theory provides two useful measure of coherence [1]. First is the l_1 norm of coherence defined as

$$C_{l1}(\hat{\rho}) = \frac{1}{d-1} \sum_{\substack{k,j \\ k\neq j}} |\rho_{k,j}|, \quad (1)$$

where $\hat{\rho}$ is $d \times d$ representation of the density matrix of the system in a given basis. The second measure of coherence is the relative entropy of coherence defined as

$$C_{\text{rel.ent.}}(\hat{\rho}) = S(\hat{\rho}_{\text{diag}}) - S(\hat{\rho}), \quad (2)$$

where S is the von Neumann entropy and $\hat{\rho}_{\text{diag}}$ represents the density matrix obtained by deleting off-diagonal elements from $\hat{\rho}$.

A typical description of neutrino oscillation involves charged current interaction in which neutrinos are produced in flavor state $|\nu_\alpha\rangle$ along with charged antilepton $\bar{l}_\alpha$. The propagation of the neutrino in the state $|\nu_\alpha\rangle$ is then governed by the propagation Hamiltonian whose eigenstates $|\nu_j\rangle$ (called mass eigenstates) follow the relation

$$|\nu_\alpha\rangle = \sum_j U_{\alpha j} |\nu_j\rangle, \quad (3)$$

where U is the unitary transformation called mixing matrix. In usual description the mass eigenstates evolve as plane waves, so that in the Schrödinger picture we can write:

$$|\nu_j(\mathbf{x},t)\rangle = e^{-iE_j t + i\mathbf{p}_j\cdot\mathbf{x}} |\nu_j(0)\rangle. \quad (4)$$

However to obtain the desired neutrino oscillation formula using the plane wave description a number of ad-hoc assumptions have to be made [3]. In addition, the plane wave treatment seems unreasonable since both neutrino production and detection are localized processes which also involve the uncertainty in the energy-momentum of neutrinos. These issues can be resolved using intermediate wave packet model, in which each propagating neutrino mass eigenstate is modeled as a wave packet. In this approach neutrino mass eignestates in momentum space are modelled as Gaussian wave packet having momentum uncertainty of production σ_p^P. Propagation of the neutrino wave packets leads to the following form of density matrix of the neutrino flavor state $|\nu_\alpha\rangle$ in the coordinate space representation [4]

$$\hat{\rho}_\alpha(x) = \sum_{k,j} U^\star_{\alpha k} U_{\alpha j} \exp\Bigg(-\frac{i\Delta m^2_{kj} x}{2E} - \Big(\frac{\Delta m^2_{kj} x}{4\sqrt{2}E^2\sigma_x^P}\Big)^2 - \Big(\frac{\Delta m^2_{kj}}{4\sqrt{2}E\sigma_p^P}\Big)^2 \Bigg) |\nu_k\rangle \langle\nu_j|, \quad (5)$$

where Δm^2_{kj} is the neutrino mass-squared difference, E is the neutrino energy, σ_x^P is the width of wave-packet in coordinate space, $\sigma_x^P \sigma_p^P = 1/2$, and x is the distance travelled by the neutrino mass-eigenstates. It can be easily seen for large distance from the production region the second term in the Eq. (5) will start to become important and the neutrinos will start to lose coherence. Thus for neutrino oscillations in vacuum we can define an effec-

*Electronic address: sjoshi@barc.gov.in

tive coherence length given by

$$L_{coh}^{jk} = \frac{4\sqrt{2}E^2\sigma_x}{|\Delta m_{jk}^2|}. \quad (6)$$

For values of distances greater than L_{coh}^{jk}, the overlap between neutrino mass-eigenstates is suppressed and the kinematic decoherence effects start to become important.

In this manuscript we will derive some relations between the coherence measures defined in Eqs. (1) and (2) and the coherence length (6) . For the simple case of two-flavor neutrino oscillations we can write down one-particle density matrix in the flavor basis as

$$\hat{\rho}_\alpha(x) = \begin{pmatrix} |U_{\alpha 1}|^2 & U_{\alpha 1}^\star U_{\alpha 2}\exp(\eta_{21}^\star) \\ U_{\alpha 2}^\star U_{\alpha 1}\exp(\eta_{21}) & |U_{\alpha 2}|^2 \end{pmatrix}, \quad (7)$$

where

$$\eta_{21} = -2\pi i\frac{x}{L_{osc}} - \Big(\frac{x}{L_{coh}}\Big)^2 - 2\pi^2\Big(\frac{\sigma_x^P}{L_{osc}}\Big)^2, \quad (8)$$

where $L_{osc} = 4\pi E/\Delta m_{21}^2$ is the vacuum oscillation length. The mixing matrix U can be parametrized by the mixing angle θ which plays an important role in determining the coherence between the two mass eigenstates. In the flavor basis the l1-norm of coherence Eq. (1) is given by

$$\begin{aligned} C_{l1}(\hat{\rho_\alpha}) =& \frac{1}{2}|U_{\alpha 1}||U_{\alpha 2}|(\exp(\eta_{21}) + \exp(\eta_{21}^\star)) \\ =& \frac{1}{2}\sin 2\theta\cos\Big(\frac{2\pi x}{L_{osc}}\Big)\exp\Big\{-\Big(\frac{x}{L_{coh}}\Big)^2 \\ & - 2\pi^2\Big(\frac{\sigma_x^P}{L_{osc}}\Big)^2\Big\}. \end{aligned} \quad (9)$$

In this case as expected C_{l1} is proportional to the mixing angle θ and is suppressed for $x > L_{coh}$.

For the second measure of coherence Eq. (2), the von Neumann entropy of a pure state can be defined as

$$S(\hat{\rho}) = -tr(\hat{\rho}\ln\hat{\rho}) = -\sum_j \lambda_j ln\lambda_j, \quad (10)$$

where λ_j are the eigenvalues of the density matrix $\hat{\rho}$ in a given basis. Now the eigenvalues of $\hat{\rho}_\alpha$ in Eq. (7) are given by

$$\lambda_{1,2} = \frac{1}{2}\Big(1 \pm \sqrt{1 - 2\sin^2\theta\cos^2\theta(1 - \exp(\eta_{21}^\star + \eta_{21}))}\Big), \quad (11)$$

where λ_1 correspond to positive sign. For the corresponding diagonal matrix $\hat{\rho}_{\text{diag}}$ the eigenvalues are

$$|U_{\alpha 1}|^2, |U_{\alpha 2}|^2. \quad (12)$$

Thus we can calculate

$$\begin{aligned} & C_{\text{rel.ent.}}(\hat{\rho}_\alpha) = -2(\cos^2\theta\ln\cos\theta + \sin^2\theta\ln\sin\theta) \\ & + \frac{1}{2}\ln\Big(2\sin^2\theta\cos^2\theta(1 - \exp(\eta_{21} + \eta_{21}^\star))\Big) \\ & + \frac{1}{2}\sqrt{1 - 2\sin^2\theta\cos^2\theta(1 - \exp(\eta_{21}^\star + \eta_{21})}\ln\frac{\lambda_1}{\lambda_2}. \end{aligned} \quad (13)$$

The main important of these coherence measure lies in their generality, since they hold true even for cases where oscillations take place in complicated matter and magnetic fields backgrounds, where coherence length does not assume a simple expression as Eq. (6). In the further work along these lines we will discuss the importance of these coherence measures and their relation to the parameters of neutrino oscillations.

References

[1] T. Baumgrarz, M. Cramer, M.B. Plenio, Phys. Rev. Lett. **113**, 140401 (2014)
[2] T. Qureshi, Quanta **8**, 24(2019)
[3] E.Kh. Akhmedov, A. Yu. Smirnov, Phys. Atom. Nucl. **72**, 1363 (2009)
[4] C. Giunti, Found. Phys. Lett. **17**, 103 (2004)