Half-off shell T-matrix study with a new approximation to nucleon-nucleon potential

P. Sahoo* and U. Laha

Department of physics, N. I. T Jamshedpur, Jharkhand-831014, India
**nlphy.pati7@gmail.com*

Introduction

The interaction between two-nucleons is basic for all of nuclear physics. The primary aim of nuclear physics is to understand the properties of atomic nuclei in terms of the exposed interaction between pairs of nucleons. With the discovery of quantum chromo dynamics (QCD), it became clear that the nucleon-nucleon (NN) interaction is not fundamental. Nevertheless, even today one assumes the nucleons to be elementary particles for nuclear structure problems. The nucleon-nucleon interaction has been investigated by a large number of researchers all over the world for the past few decades. Even though, the meson theory is not supposed to be the fundamental anymore in the light of QCD model, the meson exchange concept continues to symbolize the best working model for a quantitative nucleon-nucleon potential. The nuclear scattering of two charged particles usually occurs under the combined influence of additive interactions, one is electromagnetic in nature, and the other is nuclear in origin [1]. In general, for a short-range local nuclear plus the electromagnetic potential the Schrödinger equation does not admit an exact analytical solution. However, to obtain exact analytical solutions of the Schrödinger equation people usually replace the short-range local nuclear potential by a non-local separable one. This is no loss of generality. The major goal of the present work is to construct a non-relativistic potential that can be used easily in nuclear many-body calculations. The description of the multi-particle systems requires knowledge of the two-body interaction off the energy shell which is encountered in atomic and nuclear physics [2]. We study the off-shell Jost functions as well as half-off shell T-matrix within the nuclear Manning-Rosen plus the Hulthén potential model of interaction in all partial waves for the $(\alpha-{}^3He)$ system to examine the feasibility of our methodology. The effective nuclear Manning-Rosen plus screened atomic Hulthén potential with same range parameter is defined by

$$V_{eff}(s)=\delta^{-2}\left\{\frac{\mu(\mu-1)}{(1-e^{-s/\delta})^2}e^{-2s/\delta}-\frac{(D-E_0\delta^2)e^{-s/\delta}}{1-e^{-s/\delta}}\right\} \quad (1)$$

with $\mu=\frac{1}{2}\left[1\pm\sqrt{1+4\{\beta(\beta-1)+\ell(\ell+1)\}}\right]$ and ℓ takes the values 0,1,2,3.... Here the centrifugal barrier term is considered as $\delta^{-2}\frac{\ell(\ell+1)}{(1-e^{-s/\delta})^2}e^{-2s/\delta}$, The parameters D, β are dimensionless quantities and δ is the screening radius for nuclear potential having dimension of length with E_0, the strength.

At a centre of mass energy $E=\xi^2+i\varepsilon$ the off-shell Jost solution $f_\ell(\xi,p,s)$ for the above effective potential satisfies an inhomogeneous Schrödinger -like equation, is written as

$$\left[\frac{d^2}{ds^2}+\xi^2-V_{eff}(s)\right]f_\ell(\xi,p,s)=\left(\xi^2-p^2\right)e^{ips}. \quad (2)$$

Introducing the following transformation

$$f_\ell(\xi,p,s)=\delta^{\mu}e^{i\xi s}\left(1-e^{-s/\delta}\right)^{\mu}\Theta_\ell(\xi,p,s) \quad (3)$$

in Eq. (2) and by changing a new variable of the form $(1-e^{-s/\delta})=r$ and substituting $\mu=\eta+1$, it yields

$$r(1-r)\frac{d^2\Theta_\ell}{dr^2}+[R-(M+N+1)r]\frac{d\Theta_\ell}{dr}-MN\Theta_\ell =\delta^{2-\mu}(\xi^2-p^2)r^{\sigma-1}(1-\rho r)^{[\tau-1]}, \quad (4)$$

where $\Theta_\ell(\xi,p,s)$ is a newly defined function of momenta and position, with

$$M=1+\eta-i\xi\delta+(\eta^2+\eta+D-E_0\delta^2-\xi^2\delta^2)^{1/2},$$
$$N=1+\eta-i\xi\delta-(\eta^2+\eta+D-E_0\delta^2-\xi^2\delta^2)^{1/2},\ R=2\eta+2,$$
$$\sigma=1-\eta \text{ and } \tau=i(\xi-p)\delta. \quad (5)$$

Considering the complementary and particular solutions of Eq. (4) the complete expression for $f_\ell(\xi,p,s)$ is obtained from Eq.(3) in conjunction with Eq. (5). Utilizing the following standard integral [3]

$$\int_0^1 x^{\rho-1}(1-x)^{\sigma-1}\,{}_2F_1(\alpha,\beta;\gamma;x)\,dx=\frac{\Gamma(\rho)\Gamma(\sigma)}{\Gamma(\rho+\sigma)}\,{}_3F_2(\alpha,\beta,\rho;\gamma,\rho+\sigma;1) \quad (6)$$
$$\mathrm{Re}\,\rho>0,\ \mathrm{Re}\,\sigma>0,\ \mathrm{Re}(\gamma+\sigma-\alpha-\beta)>0$$

with consideration of boundary conditions $s\to 0$ and $s\to\infty$, $f_\ell(\xi,p,s)$ yields

$$f_\ell(\xi,p,s)=\delta^{\eta+1}e^{i\xi s}(1-e^{-s/\delta})^{\eta+1}\left[\frac{i(\xi-p)}{\Im_\ell(\xi)}\frac{\Gamma(1-\eta)\Gamma(1-i(\xi+p)\delta)}{\Gamma(1-\eta-i(\xi+p)\delta)}\right. \quad (7)$$
$${}_3F_2\left(M-1-2\eta,N-1-2\eta,-i(\xi+p)\delta;1-2i\xi\delta,1-\eta-i(\xi+p)\delta;1\right)$$
$${}_2F_1\left(M,N;R;1-e^{-s/\delta}\right)+\frac{f_\ell(\xi,p)}{\delta^{\eta+1}\Im_\ell(\xi)}(1-e^{-s/\delta})^{-2\eta-1}$$
$$\left.{}_2F_1\left(1-M^*,1-N^*;1-2i\xi\delta;e^{-s/\delta}\right)+F_p(r)\right]$$

and the on-shell Jost function $\Im_\ell(\xi)$ [4] is

$$\Im_\ell(\xi)=\delta^{\eta}\frac{\Gamma(2+2\eta)\,\Gamma(1-2i\xi\delta)}{\Gamma(M)\,\Gamma(N)}\,. \quad (8)$$

The half off-shell T-matrix is written as

$$T_{\ell h}(\xi,p,\xi^2)=\left[\frac{f_\ell(\xi,p)-f_\ell(\xi,-p)}{i\pi\, p\, f_\ell(\xi)}\right]. \quad (9)$$

The off-shell Jost function $f_\ell(\xi,p)$ is obtained from $f_\ell(\xi,p,s)$ as

$$f_\ell(\xi,p)=\lim_{s\to 0} f_\ell(\xi,p,s)$$
$$=\delta^{\eta+2}(\xi^2-p^2)\frac{\Gamma(2+\eta)\Gamma(-i(\xi+p)\delta)}{\Gamma(2+\eta-i(\xi+p)\delta)}\,. \quad (10)$$
$$\times{}_3F_2\left(M,N,\eta+2;R,2+\eta-i(\xi+p)\delta;1\right)$$

Table. 1: List of parameters for different states of $(\alpha-{}^3He)$ system.

States	[illegible]	[illegible] (fm)	[illegible]
$1/2^+$	1.89	0.671	0.05
$1/2^-$	155.892	0.384	-65.05
$3/2^-$	34.622	0.289	-16.05
$3/2^+$	1.05	0.341	0.055
$5/2^+$	1.02	0.430	0.55

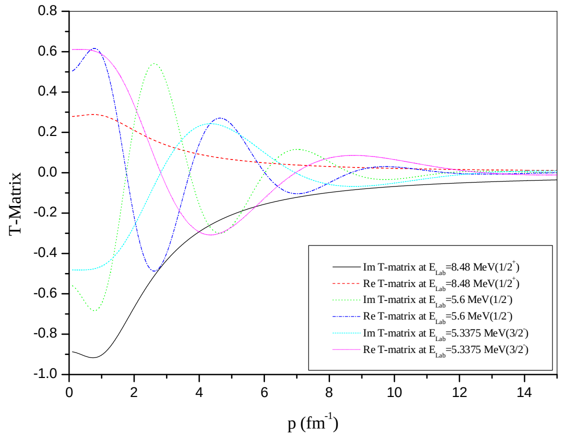

Fig 1. Half-off shell T-matrix for $(\alpha-{}^3He)$ system.

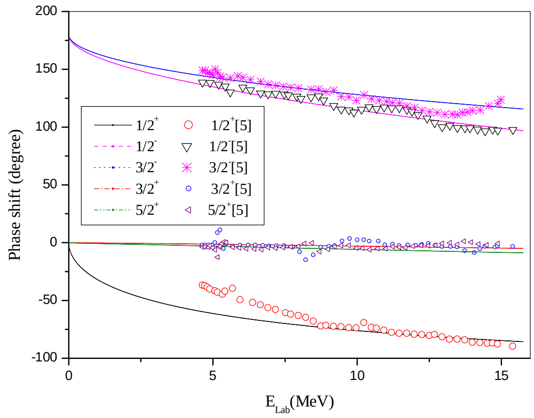

Fig 2. $(\alpha-{}^3He)$ scattering phase shifts as a function of laboratory energy.

With the parameters in Table 1 we have computed half-shell transition matrix for the $(\alpha-{}^3He)$ system using Eq. (9) which is portrayed in Fig.1. For our calculation we have used $\hbar^2/2\mu=12.0954 MeV\ fm^2$, $E_0\delta=0.4762\,fm^{-1}$ for $(\alpha-{}^3He)$ system. From Fig.1 it is seen that both $\mathrm{Re}\,T_{\ell h}(\xi,p,\xi^2)$ and $\mathrm{Im}\,T_{\ell h}(\xi,p,\xi^2)$ oscillate but approach zero as p becomes large. The function $f_\ell(\xi,p)$ also tends to zero as p increases. These indicate that the off-shell behaviour of the potential in Eq. (2) is quite acceptable. This means that the action of the potential in producing a half-off-shell T-matrix $T_{\ell h}(\xi,p,\xi^2)$ depends also on p. It is well known that the phase of the half-shell transition matrix is the scattering phase shift. The set of curves for $\mathrm{Re}\,T_{\ell h}(\xi,p,\xi^2)$ and $\mathrm{Im}\,T_{\ell h}(\xi,p,\xi^2)$ satisfy this criterion. The phase parameters calculated from the half-shell T-matrix are depicted in Fig. 2 which are in reasonable agreement with those of Spiger and Tombrello [5].

References

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