# Structural analysis of the exotic $^{40}$Mg nucleus at the neutron dripline

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## Introduction

Nuclear physicists are paying close theoretical as well as experimental attention since the advent of the radioactive ion beam facility (RIBF) of RIKEN Nishina Centre to one of the most significant topics in the study of nuclear structure of exotic nuclei that are acknowledged to be beyond the spherical shape. Nuclear driplines are the limits beyond which a single nucleon towards the nuclear ground state becomes unbound, and they constitute the margins of the nuclear chart. A dense stable core is surrounded by a low-density envelope with a long extension in this fascinating nuclear structure. A Literature study unveils that One of the most exotic light neutron-rich nuclei currently available for experimental research is $^{40}$Mg [1], which lies at the intersection of the nucleon magic number N=28 and the neutron dripline. F. G. Kondev et al (2021) [2], revealed that $S_{2n}$ of $^{40}$Mg is $0.670 \pm 0.71$ MeV using the most recent atomic mass evaluation (AME 2020). H. L. Crawford et al (2014) [3], stated theoretically for $^{40}$Mg the enhancement of matter radii (approximately 3.6 fm), $S_{2n} = 1.119$ MeV, and second low energy transmission at 670(16) KeV. $^{40}$Mg likely possesses two neutrons in the $low-l$ $2p_{3/2}$ orbital at the Fermi surface, according to Nilsson and Shell model calculations by E. Caurier (2014) [4] and I Hamamoto (2016) [5].

In the present work, we will investigate the ground and resonance states of $^{40}$Mg in the framework of the few-body model using a classy theoretical scheme. We assume a three-body model of $^{40}$Mg as a structureless core $^{38}$Mg surrounded by two valence neutrons (n). We first solve for the bound state of the three-body system using standard GPT [6] n-n potential and standard SBB [7] core-n potential. Parameters of the core-n potential are going to be adjusted to make sure that $^{39}$Mg subsystem is simply unbound.

The core-n potential parameters were chosen based on the fact that the $^{39}$Mg subsystem is completely unbound. The one-parameter family of isospectral potentials was then constructed using the ground state wave function. The parameter is tuned to create a deep well, followed by a positive barrier that aids in particle trapping within the deep well, and a sharp barrier at energy E($>0$). The probability of the particle being trapped within the well-barrier combination is calculated for various positive energies, with a peak at resonance energy. We utilized the WKB approximation to calculate the resonance width after identifying the resonance energy.

## Method

A three-body $^{40}$Mg unusual nuclear system is solved using the Hyperspherical Harmonics Expansion Method. The much heavier nuclear core $^{38}$Mg is designated as particle 'i' whereas two orbiting valence nucleons are designated as particles "j" and "k," respectively. The Jacobi Cordinates are defined as follows in the partition 'i' with i,j,k = 1,2,3 cyclic: Recently, Khan et al (2021)[8], successfully tested the present method for Gaussian type potential to the core plus two-neutron three-body model.

$$\vec{x_i} = a_i(\vec{s_j} - \vec{s_k}) \tag{1}$$

$$\vec{y_i} = \frac{1}{a_i}\left(\vec{s_i} - \frac{m_j\vec{s_j} + m_k\vec{s_k}}{m_j + m_k}\right) \tag{2}$$

$$\vec{R} = \frac{(m_i\vec{s_i} + m_j\vec{s_j} + m_k\vec{s_k})}{M} \tag{3}$$

where $a_i$ is const. ; $m_i, \vec{s_i}$ are the mass and position of the $i^{th}$ particle and $M = m_i + m_j + m_k, \vec{R}$ is the centre of mass (CM) of the system. The hyperradius $\rho$, an invariant under three dimensional rotations and permutations of particle indices together with the five angular variables $\Omega_i \rightarrow \{\phi_i, \theta_{x_i}, \phi_{x_i}, \theta_{y_i}, \phi_{y_i}\}$ constitute hyperspherical variables of the system. It should be noted that hyperangles $\Omega i$ are determined by the partition "$i$" chosen. The Schrödinger equation is rewritten in terms of hyperspherical variables $(\rho, \Omega_i)$:

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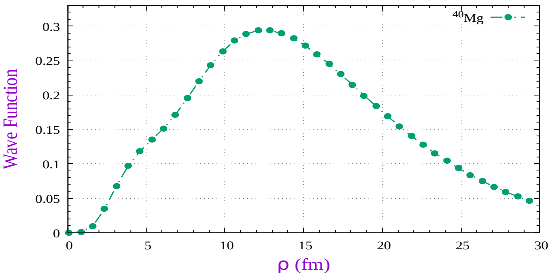
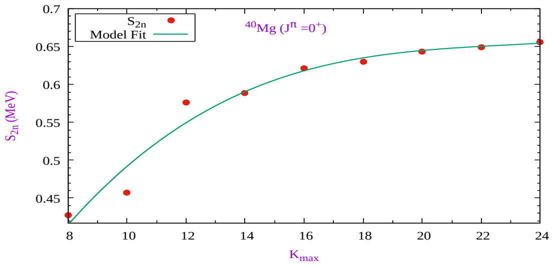
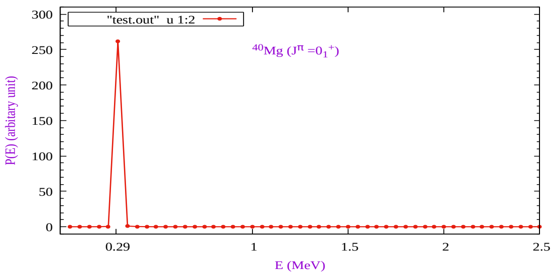



(a) Represention of wave function of $^{40}$Mg.




(b) Binding Energy vs Kmax graph of $^{40}$Mg




(c) Represention of resonant state of $^{40}$Mg.

$$\left[-\frac{\hbar^2}{2\mu}\left\{\frac{1}{\rho^5}\frac{\partial}{\partial\rho}(\rho^5\frac{\partial}{\partial\rho}) - \frac{\hat{\mathcal{K}}^2(\Omega_i)}{\rho^2}\right] + \\ \left[V(\rho,\Omega_i) - E\right]\Psi(\rho,\Omega_i) = 0 \tag{4}$$

where $V(\rho, \Omega_i)$ is the total interaction potential in the partition "$i$", and the eigenvalue equation is satisfied by the square of the hyper angular momentum operator $\hat{\mathcal{K}}^2(\Omega_i)$ .

## Results and Discussions

In this article, we used a novel technique to investigate resonances in nuclei with at least one bound state as well as those without a bound ground state. For this purpose, the Super Symmetric Quantum Mechanics (SSQM) formalisim will be applied to the Hyperspherical Harmonic Expansion Method (HHEM) for few-body nuclear systems such as $^{40}$Mg. In the case of weakly bound exotic systems, solving the three-body Schrodinger equation to produce a completely converged solution is extremely challenging. This method, on the other hand, could be highly useful for studying weakly bound states, as well as bound states in continuum and resonances. The HHE Method used here is an essentially accurate method in which the binding energies steadily converge as Kmax values increase. Results are summarized in Table 1, indicates $^{40}$Mg possible 2-neutron halo candidate.

TABLE I: **Comparison of the calculated results with experimental data and other reference works found in the literature for $^{40}$Mg.**

| $K_{Max}$ | $S_{2n}$ (MeV) | $P_{lx=0}$ | $E_{lx=0}$(MeV) |
|---|---|---|---|
| 8 | 0.4272 | 0.7787 | -0.3052 |
| 12 | 0.5761 | 0.7863 | -0.4262 |
| 16 | 0.6210 | 0.7901 | -0.4652 |
| 20 | 0.6437 | 0.7927 | -0.4858 |
| 24 | 0.6562 | 0.7942 | -0.4967 |
| ... | | | |
| $\infty$ | 0.6756 | | |
| State | Observables | Present work $S_{2n}$ | Others work |
| $0^+$ | $S_{2n}$ (MeV) | 0.6756 | 0.670 ±0.71[2] |
| | $R_A$ (fm) | 3.63 | 3.60[3] |
| $0_1^+$ | $E_R$(MeV) | 0.29 MeV | - |
| | Γ(MeV) | 0.478 MeV | - |

## Acknowledgments

Authors would like to express an appreciation of necessary facilities from Aliah University.

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