Off-Shell T-matrix for the Manning-Rosen Potential

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Introduction

T-matrix has an importance in the literature of the two-nucleon scattering theory as one can virtually obtain all the relevant scattering quantities directly from these matrix elements. The T-matrices are equally important while dealing with three particle scattering/reaction. The two particle T-matrices work as the basic ingredients in the integral equations of Faddeev [1] and other equations developed later and are source of various calculations as its on-the energy shell, Half-shell and fully Off-shell versions [2, 3] define different physical aspects of the same scattering phenomenon.

It is of importance to have analytical expressions of off-shell Jost solution, physical solution as well as T-matrices in the literature associated with the scattering using the Manning-Rosen Potential what has quite frequently been used in atomic and molecular physics. However, we use the Manning-Rosen potential in the realm of nuclear scattering in this work. In this write up, we present the analytical expression for the physical solution and calculate off-shell T-matrix using an expression that does not explicitly depend on the potential. We take the n-p system scattering as a case study for our T-matrices in different states.

Physical solution and T-matrix

The s-wave off-shell physical solution satisfies the following standard differential equation

$$\left[\frac{d^2}{dr^2}+k^2-\frac{1}{b^2}\left\{\frac{\alpha(\alpha-1)e^{-2r/b}}{\left(1-e^{-r/b}\right)^2}-\frac{Ae^{-r/b}}{\left(1-e^{-r/b}\right)}\right\}\right]\psi_m^{(+)}(k,q,r)=\left(k^2-q^2\right)\sin qr \tag{1}$$

The particular solution to the above equation is written as

$$\psi_m^{(+)}(k,q,r)=\left(k^2-q^2\right)\int_0^\infty dr'\sin qr' G_m^{(+)}(r,r') \tag{2}$$

with

$$\overline{G}_m^{(+)}(r,q)=\int_0^\infty dr'\, G_m^{(+)}(r,r')e^{iqr'} . \tag{3}$$

The expression for the physical green function reads as

$$G_m^{(+)}(r,r')=-\frac{1}{k}\psi_m^{(+)}(k,r_<)f_m(k,r_>), \tag{4}$$

where $\psi_m^{(+)}(k,r)$ stands for the on-shell physical solution of the Manning-Rosen potential, given by

$$\psi_m^{(+)}(k,r)=\frac{k\phi_m(k,r)}{f_m(k)} \tag{5}$$

with $\phi_m(k,r)$ being the regular solution of the Manning-Rosen potential. For T-matrix, the analytical expression without depending on the potential explicitly, reads as

$$T\left(p,q,k^2\right)=\frac{2\left(k^2-p^2\right)}{\pi pq}\int_0^\infty dr\sin(pr)\psi^{(+)}(k,q,r) . \tag{6}$$

Utilizing the expression for $\psi^{(+)}(k,q,r)$, the above can further be extended as follows

$$T\left(p,q,k^2\right)==\frac{\left(k^2-p^2\right)\left(q^2-k^2\right)}{2\pi pq}\times$$
$$\left\{T_1\left(p,q,k^2\right)-T_2\left(p,-q,k^2\right)-T_3\left(-p,q,k^2\right)+T_4\left(-p,-q,k^2\right)\right\} \tag{7}$$

where

$$T_1\left(p,q,k^2\right)=\int_0^\infty dre^{ipr}\overline{G}_m^{(+)}(r,q) .$$

Expressions of $T_2\left(p,-q,k^2\right), T_3\left(-p,q,k^2\right)$ and $T_4\left(-p,-q,k^2\right)$ can be obtained from the expression of $T_1\left(p,q,k^2\right)$ with substitutions of q by ï q, p by ï p and p & q by ï p &-q. Thus we obtain the closed form analytical expression for the T matrix. We have also made few of the useful checks on our expression of T-matrix with respect to its on-shell, half-shell and Hulthén limit [4] and found to reproduce correct limiting results.

Results and Discussion

We study the n-p system for its $^1S_0$, $^3S_1$ states with laboratory energies of 10 and 20 MeV. For the n-p system we have used $\hbar^2 / m_p = 41.47\, MeV\ fm^2$. To compute the off-shell T-matrices for (n-p) system we use the parameters A=0.952, b=1.152 fm, α = -0.0043 for $^1S_0$ and A=1.57, b=1.2135 fm, α =0.005 for $^3S_1$ states respectively.

For the laboratory energy, 10 MeV with q=0.55 fm^{-1}, Re T(p,q,k^2) initially decreases up to p=0.5 fm^{-1} and beyond that point increases smoothly to zero as can be seen from figure.1. For Lab energy of 20 MeV, Re T(p,q,k^2) decreases up to p=0.3 fm^{-1} at the beginning and then increases whereas Im T(p,q,k^2) increases from a negative value with off-shell momentum p approaching towards zero.

For $^3S_1$ state with q=0.55 fm^{-1} , Re T(p,q,k^2) at E_{Lab}=10 MeV reaches its lower peak at p=0.6 fm^{-1} and for E_{Lab}=20 MeV it shows small oscillations in between p=0.4-0.9 fm^{-1}. The Im T(p,q,k^2)s are smooth increasing function of p. The Figure 2 presents the T-matrix values for the (n-p) **^{3}S$_1$** state.

Conclusions

The present work deals with the (n-p) system with the Manning-Rosen potential which is a simple two-nucleon model potential in form with three adjustable parameters while the standard models have lot many parameters to be fitted. Simple potential models are good enough to reproduce low energy scattering parameters for two-nucleon systems and do not fit high energy data. This simple minded potential model has the right ability to reproduce correct nature of the off-shell behavior of two-nucleon system. It is our belief that the present work may be of quite interesting to a wide variety of nuclear physicists.

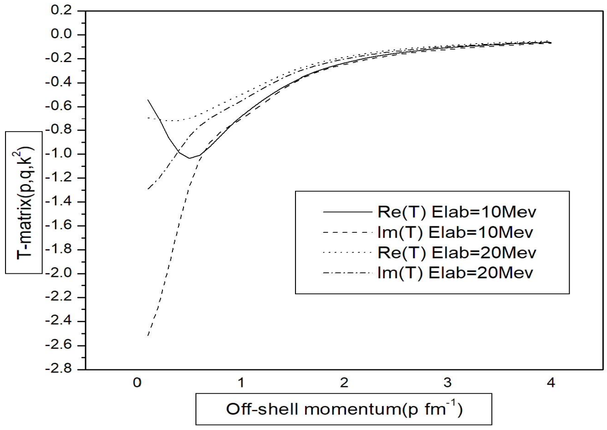

Fig.1 T-matrix as function of p for (n-p) $^1S_0$ state

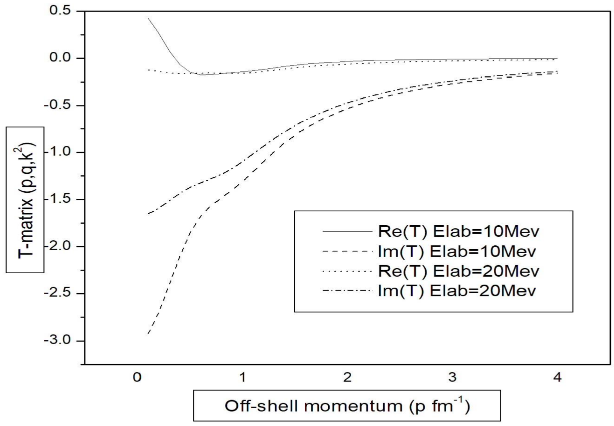

Fig.2 T-matrix as function of p for (n-p) $^3S_1$ state

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