

¹¹Be (One-neutron halo) + ²⁰⁹Bi reaction in multi-body three-stage Classical Molecular Dynamics model

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Introduction

¹¹Be has well established one-neutron halo structure having low valance neutron separation energy ($S_n = 0.501 \text{ MeV}$) [1]. Due to the low separation energy of valance neutron, this neutron has a relatively high probability of being knocked out. Such breakup effects can be better investigated with a heavier target (i.e. ²⁰⁹Bi) due to the increasing predominance of long-range Coulomb interaction compared to the nuclear potential [2].

Reactions involving such weakly bound nuclei can be studied using multi-body 3-Stage Classical Molecular Dynamics (Multi-body 3S-CMD) model [3] in which the dynamical collision simulation is carried out in three stages, (1) Rutherford trajectory calculation (2) Classical Rigid Body Dynamics (CRBD) calculation [4] and (3) Classical Molecular Dynamics (CMD) [5] calculations.

For the multi-body 3S-CMD model, ¹¹Be is constructed as a cluster of tightly bound ¹⁰Be and one valance neutron. Here, tightly bound ¹⁰Be is constructed using variational potential energy minimization STATIC [5] code and “cooled” using DYNAMIC [5] code. As some other weakly bound projectiles already constructed in [3], using a “dynamic cooling” method by carrying out rigid body dynamics like procedure and setting the cluster velocities and their angular moment zero after every time-step and thus obtaining the equilibrium orientation and position of the centre of mass of these constituents (¹⁰Be+n). Now, the distance between the centre of mass of ¹⁰Be and neutron is adjusted in such a way that the typical ion-ion potential between them is equal to the experimental neutron separation energy of ¹¹Be.

In this contribution, by systematically removing rigidity constraints on the target ²⁰⁹Bi,

the projectile ¹¹Be(¹⁰Be+n) complete fusion cross-sections are calculated. This model can also account for a direct reaction like a neutron transfer from the projectile in this case. Therefore, the probabilities of different events as a function of E_{CM} can also be studied for this reaction.

Model Details

For the construction of ¹¹Be(¹⁰Be+n) and ²⁰⁹Bi, a purely phenomenological softcore Gaussian Potential is used given by,

$$V_{ij}(r_{ij}) = -V_0 \left(1 - \frac{C}{r_{ij}} \right) \exp \left(-\frac{r_{ij}^2}{r_0^2} \right) \dots \dots (1)$$

Where, V_0 , C , and r_0 are respectively, the depth parameter, repulsive-core radius and range parameter.

Here, the potential parameter set $V_0 = 710.0 \text{ MeV}$, $C = 1.88 \text{ fm}$ and $r_0 = 1.15 \text{ fm}$ is used to produce ground state properties of ²⁰⁹Bi and ¹¹Be and are mentioned in Table 1. The nucleons of constituent nuclei of the nucleus are not allowed to move within respective nuclei.

Table 1: Ground-state properties

Nucleus	Calculated		Experimental	
	B.E. (MeV)	R (fm)	B.E. (MeV) [6]	R (fm)
¹⁰ Be	59.68	2.09	65.97	2.28 [7]
¹¹ Be (¹⁰ Be+n)	60.18	2.06	65.55	2.90 [7]
²⁰⁹ Bi	1606.6	5.55	1640.26	5.52 [8]

In the 3S-CMD model, during dynamical collision simulation, for the second stage, the

target and projectile are treated as complete rigid bodies. This rigid body constraint can be systematically removed in the third stage from the appropriate projectile-target separation distance.

We have carried out simulations for three different cases, (1) SBPM [9] calculation, in which all dynamical effects are neglected, (2) $^{11}\text{Be}(\text{R})+^{209}\text{Bi}(\text{NR})$, treating ^{11}Be completely rigid and ^{209}Bi non-rigid in the third stage, and (3) $^{11}\text{Be}(^{10}\text{Be}(\text{R})\text{-NR-}n)+^{209}\text{Bi}(\text{R})$, treating ^{10}Be rigid, the bond between ^{10}Be and neutron non-rigid and ^{209}Bi non-rigid in the third stage to calculate fusion cross-sections.

Here, fusion cross sections for collision energy E_{cm} is calculated using Wong's formula [10],

$$\sigma_{fus}(E_{cm}) = \left[\frac{R_B^2 \hbar \omega_0}{2E_{cm}} \right] \ln \left[1 + \exp \left(2\pi \frac{(E_{cm} - V_B)}{\hbar \omega_0} \right) \right] \dots (2)$$

Result and discussions

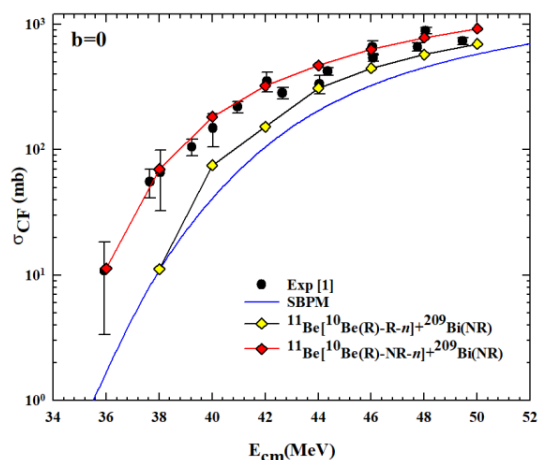


Fig.1 Complete fusion cross sections for the $^{11}\text{Be}+^{209}\text{Bi}$ reaction

From the cross-section calculation for the cases mentioned above, it is evident that in SBPM calculation, where all dynamical effects are completely neglected, the fusion cross-section is suppressed. When we remove rigidity constraint on the target in the third stage of multibody 3S-CMD model calculations, fusion cross-section is

enhanced compared to SBPM but it is not in agreement with the experimental data.

For the third case, when we relaxed the bond between ^{10}Be and neutron in ^{11}Be , there is an enhancement of fusion cross-section and it is also in good agreement with the experimental data.

It can be understood that systematic relaxations of constraint in multibody 3S-CMD model calculation help us understand the effects of different degrees of freedom on the outcome of the calculation.

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