On the distance of closest approach in one-dimensional Coulomb scattering

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Introduction

The alpha particle scattering experiment is a milestone in the development of modern physics [1, 2]. The early explanation of the scattering pattern by Ernest Rutherford led to the discovery of atomic nuclei. Over time this technique has found many applications, especially in nuclear physics and material science. In Ref. [3, 4], we investigated the emergence of nonclassicality in the one-dimensional version of this experiment, and here we summarize the implications on the 'distance of closest approach' of the projectile.

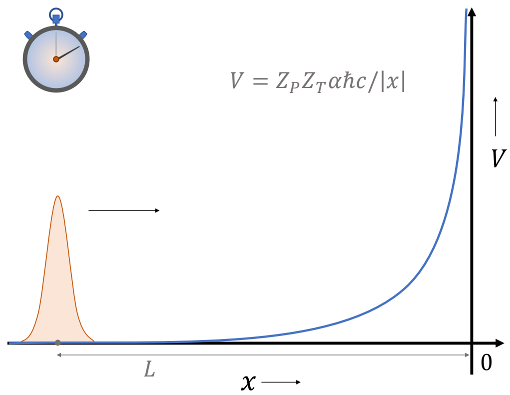

FIG. 1: Model of the experiment [3, 4]. The projectiles are prepared as Gaussian wave packets, which evolve in the Coulomb potential of the target nucleus fixed at the origin. The symbols have their usual meanings.

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Theoretical Framework

We model the simplest case of Rutherford scattering, i.e., the head-on collision of alpha particles with stationary gold nuclei. The projectile-target interaction is considered as due to the Coulomb potential only:

$$V(x) = Z_P Z_T \alpha\hbar c/|x|, \tag{1}$$

where the target is fixed at the origin. Z_P and Z_T are the charges carried by the projectile and the target, respectively, and α is the fine-structure constant. We prepare the projectile in a Gaussian wave packet centered at the separation L with a width σ and an average momentum $P = \sqrt{2mT_0}$. The resulting quantum trajectories are compared with those of a classical alpha particle with kinetic energy T_0. The time-evolution of the quantum state is calculated using the Cayley's form of the evolution operator [5]:

$$U(\Delta t) = \left(1 + i\frac{H\Delta t}{2\hbar}\right)^{-1}\left(1 - i\frac{H\Delta t}{2\hbar}\right), \tag{2}$$

where H is the Hamiltonian.

Results

The convexity of Coulomb interaction implies that the expected quantum force can be very different from the force experienced in the classical theory. This leads to many consistent deviations between Ehrenfest's and Hamilton's dynamics [3, 4]. Here we discuss the impacts on the minimum of projectile-target separation, i.e., the distance of closest approach. In the classical theory this is set solely by the Coulomb potential:

$$d_{\rm cl} = \frac{Z_P Z_T \alpha \hbar c}{T_0 + V(L)}. \qquad (3)$$

In the quantum theory, this is decided by the Coulomb interaction together with Jensen's inequalities and the laws of quantum mechanical evolution.

As an illustration, at $t = 0$, we can use the Jensen's inequality to write:

$$|\langle F \rangle| \sim \left\langle \frac{1}{x^2} \right\rangle \geq \frac{1}{\langle x \rangle^2} = \frac{1}{x_{\rm cl}^2} \sim |F_{\rm cl}|, \qquad (4)$$

where F is the force. This implies that the quantum wave packet experiences a stronger repulsion, and hence it must be reflected from a larger separation compared to its classical counterpart. Fig. 2 shows that this is indeed the case for each and every quantum configuration. The existence of global minimas also proves that there is a well defined limit on the classical-quantum agreement, and contrary to the popular belief, the classical solutions are never recovered [6]. These minimas are very well approximated to occur at the initial standard deviation given by $\sigma_0 = \sqrt{\hbar L/2P}$, and a comprehensive study of such configurations

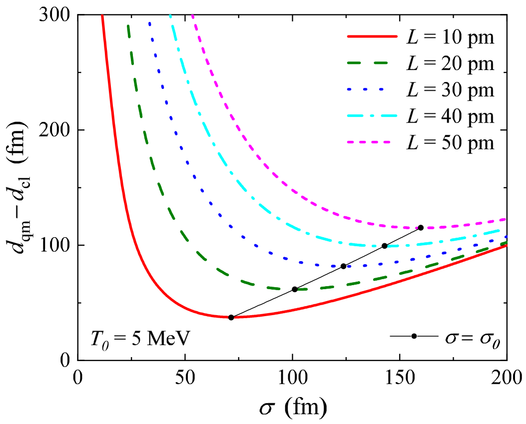

FIG. 2: Difference between the classical and quantum distances of closest approach [3, 4]. The symbols have their usual meanings and are explicitly explained in the main text.

constrains the 'quantum distance of closest approach' by [3, 4]

$$d_{\rm cl} < d_{\rm qm} < d_{\rm cl} + \sqrt{\frac{\hbar L}{2P}}. \qquad (5)$$

Conclusions

The uncertainty principle implies a nonzero momentum variance, and hence a nonzero kinetic energy, of practically any particle. The differences are therefore expected between classical and quantum dynamics in situations where the classical particle stops. In Ref. [3, 4], we have computed a number of such differences for the case of central collision between a charge in motion and another stationary charge, and here we summarized the implications on the distance of closest approach. Additionally to the uncertainty principle an important element of physical understanding of this scenario is the convexity of Coulomb interaction.

Acknowledgements

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