Nuclear Reactions at Low Energies in Condensed Medium

P. Jain[1,*] H. Kumar[1], K. Ramkumar[1], A. Kumar[1], R. Pala[2], and K. P. Rajeev[1]
[1]*Department of Physics, Indian Institute of Technology, Kanpur, 208016, India and*
[2]*Department of Chemical Engineering, Indian Institute of Technology, Kanpur, 208016, India*

Introduction

There have been many claims that nuclear reactions can take place at low energies in a condensed medium [1]. Experimentally there exist claims of excess heat, nuclear transmutations as well as production of particles which can only arise from a nuclear phenomenon. A recent detailed study [2] finds that while it is difficult to confirm excess heat, the nuclear reaction rate appears to be significantly enhanced at low energies in comparison to what may be theoretically expected. Theoretically there have been many attempts to understand this phenomenon but there does not exist any universally accepted model.

It has been suggested that the low energy nuclear processes might take place at second order in time dependent perturbation theory [3–5]. Here we investigate this possibility by considering a particular process [6],

$$ {}^2\mathrm{H} + {}^A\mathrm{X} \to {}^{A+1}\mathrm{X} + {}^1\mathrm{H} + \gamma(\omega_1) + \gamma(\omega_2)\,. \quad (1)$$

Here ^{A}X is a nucleus of mass number A and we will assume that A is relatively large. In this process a deuteron nucleus reacts with a heavy nucleus ^{A}X to produce an isotope ${}^{A+1}$X with emission of a proton and two photons. It happens through a capture of neutron by the nucleus ^{A}X. To be specific, here we shall consider X to be the nucleus ^{58}Ni. A detailed calculation of this process is presented in [6]. Here we describe the main features of the calculation and the physical picture that emerges.

Physical Mechanism

The process shown in Eq. 1 is expected to be heavily suppressed due to Coulomb repulsion if the initial state energy is small. However at second order in perturbation theory, there is some possibility to overcome the Coulomb barrier since all energy eigenstates of the system contribute to the process [3–5]. We see this from the basic equation for the transition amplitude at this order [7],

$$\begin{aligned} \langle f|T(t_0,t)|i\rangle &= \left(\frac{i}{\hbar}\right)^2 \sum_n \int_{t_0}^{t} dt' e^{i(E_f-E_n)t'/\hbar} \\ &\quad \langle f|H_I(t')|n\rangle \int_{t_0}^{t'} dt'' e^{i(E_n-E_i)t''/\hbar} \\ &\quad \langle n|H_I(t'')|i\rangle\,. \end{aligned} \quad (2)$$

where $|i\rangle$, $|n\rangle$ and $|f\rangle$ are the initial, intermediate and final states respectively and H_I is the interaction Hamiltonian. Here E_i, E_n and E_f are energies of the initial, intermediate and final states respectively. The interaction Hamiltonian is given by [7]

$$H_I(t) = \sum_i \frac{Z_i e}{c m_i}\vec{A}(\vec{r}_i,t)\cdot\vec{p}_i + \sum_i \frac{e g_i}{2 m_i c}\vec{S}_i\cdot\vec{B}(\vec{r}_i,t) \quad (3)$$

where $\vec{A}$ is the electromagnetic field operator, $\vec{B} = \vec{\nabla}\times\vec{A}$ the magnetic field and $\vec{S}_i$ the spin operator of the ith particle. Furthermore, $Z_i e$, m_i, $\vec{r}_i$ and $\vec{p}_i$ are respectively the charge, mass, position vector and momentum vector of the ith particle.

We assume that in the initial state deuteron and ^{58}Ni form a spherically symmetric molecular bound state. The wave function of this state depends on the medium conditions and we assume it to be a Gaussian centered at R_0. The heavy nucleus is assumed to be at the origin of our coordinate system and R_0 is a measure of the effective distance between the proton-neutron system and the heavy nucleus in the initial and the intermediate states. After the application of the first perturbation

*Electronic address: pkjain@iitk.ac.in

the nuclear system emits a photon of energy $E_1 = \hbar\omega_1$ and makes a transition to an intermediate state, which can be expressed as a linear superposition of all eigenstates of the unperturbed Hamiltonian, irrespective of the energy of the eigenstate. Hence energy is not conserved at this vertex. However the contribution from high energy states is suppressed by a power of $\Delta E = E_n - E_i + E_1$. It turns out that this suppression is relatively mild compared to the suppression due to Coulomb repulsion. We point out that, although energy is not conserved at each vertex, overall energy is conserved. Furthermore, momentum must remain conserved at each vertex, failing which the process is exponentially suppressed. The momentum conservation is a little involved in our case due to the presence of a potential.

A detailed calculation of the process in Eq. 1 is given in [6]. For a model calculation, we use a standard textbook nuclear potential for the deuteron and the ^{58}Ni nucleus. At the first interaction the proton-neutron system makes a transition from the $l = 0$ ground state to an $l = 1$ intermediate state involving a free proton and a neutron through the action of the first term in the interaction Hamiltonian, Eq. 3. In the second interaction, the free neutron gets absorbed by the heavy nucleus. We treat this by considering the magnetic moment term in Eq. 3. Here we are interested in only demonstrating that the process is feasible for which this is sufficient. For a detailed calculation both terms in Eq. 3 need to be considered. Due to the presence of the heavy nucleus, the neutron wave function in the neighbourhood of the heavy nucleus gets modified. We consider only the dominant spherically symmetric part of this modification to compute the matrix element $\langle f|H_I(t')|n\rangle$. The final state also involves an emitted proton whose wave function is taken to be a plane wave. The proton wave function would be heavily suppressed in the vicinity of the heavy nucleus and it is important to include this correction to its wave function. However for the present calculation the matrix element involving the final state proton is heavily suppressed in the vicinity of ^{58}Ni due to Gaussian center of mass wave function of the proton neutron system. Hence this region contributes negligibly and we need not worry about this modification to proton wave function in the calculation of this matrix element. We point out that only the $l = 1$ part of the proton wave function contributes.

A detailed calculation of the resulting process reveals that the process gives appreciable rates only if the final state proton energy is relatively small, of the order of an eV or less. The reason for this is physically clear. By momentum conservation, it is only in this region that the intermediate state energy, and hence the neutron energy, can be of the order less than an eV. With such low energies, the neutron de Broglie wave length can be sufficiently large to be comparable to R_0, which is a measure of the distance between deuteron and ^{58}Ni in the initial state. We find that the rates are small, but observable in a condensed medium. The reaction leads to two photon emission in coincidence and can be tested cleanly in a laboratory.

References

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