

## Magnetic deformation in Neutron Stars

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### Introduction

Heavy neutron stars (NSs) are expected to contain exotic matter in their interior, even if they are rotating fast [2]. Very massive and/or fast rotating stars could be the result of accretion, or even a previous stellar merger, both of which have been shown to enhance stellar magnetic fields [2]. With exceptionally high density, a magnetic field reaching  $\approx 10^9$  to  $10^{18}$  G [3] is attainable in massive NS centers.

Previous works have studied the effect of the magnetic field on the NS EoS and the stellar properties like mass and radius [4]. The presence of a strong magnetic field deviates the NS structure from spherical symmetry of the strongly and hence the spherically symmetric Tolman–Oppenheimer–Volkoff (TOV) equations can no longer be applied for studying their macroscopic structure [5].

In the present work, we employ recently proposed density-dependent Relativistic Mean-Field (DD-RMF) parameter set DD-MEX to study the properties of NSs. By reproducing different hyperon-hyperon optical potentials, the values of the hyperon couplings are obtained, using several different coupling schemes, which are then used to model hyperons in our calculations. We further analyze the effect of magnetic fields on the nucleonic and hyperonic matter by employing a realistic chemical potential-dependent magnetic field [5].

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### Formalism

The total Lagrangian density for the DD-RMF model in the presence of a magnetic field  $B$  is given by [5, 6]. The total energy density in presence of the magnetic field  $B$  is

$$\mathcal{E} = \mathcal{E}_m + \frac{B^2}{8\pi}. \quad (1)$$

The total pressure in the perpendicular to the local magnetic field is

$$P_{\perp} = P_m - \mathcal{M}B + \frac{B^2}{8\pi}, \quad (2)$$

where  $\mathcal{M}$  is the magnetization. The total pressure in the parallel direction to the local magnetic field does not contain the magnetization term.

### Results and Discussion

For the strength of the local field, we employ a chemical potential-dependent profile which was derived from the solutions of the Einstein-Maxwell equations. It is given by [5]

$$B^*(\mu_B) = \frac{(a + b\mu_B + c\mu_B^2)}{B_c^2} \mu, \quad (3)$$

with  $\mu_B$  being the baryon chemical potential in MeV and  $\mu$  the dipole magnetic moment in units of  $\text{Am}^2$  to produce  $B^*$  in units of the electron critical field  $B_c = 4.414 \times 10^{13}$  G. The coefficients  $a$ ,  $b$ , and  $c$  are obtained from a fit for the magnetic field in the polar direction of a star with a baryon mass of  $2.2M_{\odot}$ .

The values of the magnetic field produced at the surface and at large densities using different values of the magnetic dipole moment  $\mu$

TABLE I: Magnetic field at stellar surfaces  $B_s$  and at center  $B_c$  for DD-MEX EoS at  $2.2M_\odot$  baryonic mass for an NS and an hyperon star (HS).

$\mu$ ( $\text{Am}^2$ )	Neutron Star		Hyperon Star	
	$B_s$ ( $10^{16}\text{G}$ )	$B_c$ ( $10^{17}\text{G}$ )	$B_s$ ( $10^{16}\text{G}$ )	$B_c$ ( $10^{17}\text{G}$ )
$5 \times 10^{30}$	0.101	0.259	0.665	0.196
$5 \times 10^{31}$	8.98	2.28	5.83	1.89
$10^{32}$	1.79	4.55	1.12	3.77

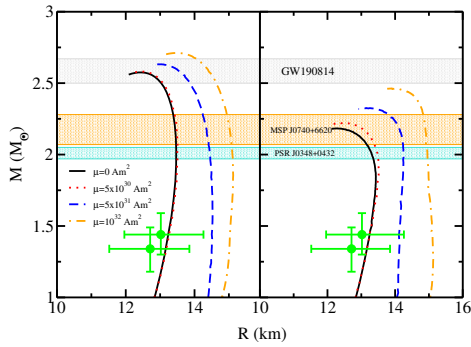


FIG. 1: Relation between mass and circumferential radius for an NS without hyperons (left panel) and with hyperons (right panel) using LORENE.

for NSs with and without hyperons are shown in Table I.

Fig. 1 displays the mass-radius relation for DD-MEX parameter set for NS and HS with and without magnetic field using LORENE. The NS maximum mass without hyperons increases from  $2.575M_\odot$  for  $B = 0$  to  $2.711M_\odot$  for  $\mu = 10^{32} \text{ Am}^2$  in the perpendicular direction. The corresponding radius changes from 12.465 to 13.474 km. The radius at  $1.4M_\odot$  increases by around 1 km. With the hyperons included, the mass increases from 2.183 to  $2.463 M_\odot$  when magnetic field effects are included with  $\mu = 10^{32} \text{ Am}^2$ . The variation obtained in the mass-radius is larger for hyperonic stars than for the pure nucleonic stars due to the additional effect of de-hyperonization that takes place due to the magnetic field.

Fig. 2 displays the mass-radius profile same as Fig. 1 by solving general relativity spherically symmetric solutions (TOV) using  $P_\perp$ .

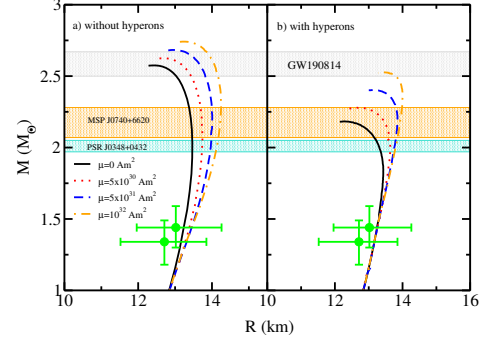


FIG. 2: Relation between mass and radius for an NS without hyperons (left panel) and with hyperons (right panel) solving TOV for a static NS.

As the magnetic field increases by changing magnetic dipole moment, the maximum mass increases by about  $0.1M_\odot$  for NS and  $0.2M_\odot$  for HS. The change in the radius at canonical mass is very small. Even with the strong magnetic field produced by magnetic dipole moment  $\mu = 10^{32} \text{ Am}^2$ , the radius satisfies all the constraints. Thus, we see that neglecting the deformation effects by solving the spherically symmetric TOV equations, leads to an overestimation of the mass and an underestimation of the radius. This happens because the extra magnetic energy that would deform the star is being added to the mass due to the imposed spherical symmetry.

## References

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