.

Mass Spectra of Charmonium and Bottomonium Using the Coulomb Perturbed Potential

Vinod Kumar[1], S. B. Bhardwaj[2], Ram Mehar Singh[1], and Fakir Chand[3*]
[1]*Department of Physics, Ch. Devi Lal University, Sirsa - 125055, INDIA*
[2]*Department of Physics, S. U. S. Govt. College, Indri, Karnal - 132041, INDIA and*
[3]*Department of Physics, Kurukshetra University, Kurukshetra - 136119, INDIA*

Introduction

In meson spectroscopy, the properties of heavy and light meson are of great interest. With the help of a Schrodinger equation (SE), one can predict various properties of mesons by the non-relativistic potential models using different methodology. It is generally believed that the quarkonia, which are considered as bound states of a quark and an anti-quark are essentially non-relativistic. Therefore, in order to estimate various properties of mesons, a potential model based approach has been quite successful [1]. Also, a theoretical study should be able to predetermine the decay properties in addition to mass spectra. So in present work we employ the Coulomb perturbed potential (CPP) [2] to predict the mass spectra of various mesons and compare the results with available experimental and theoretical outcomes. Here the interaction potential used is

$$V(r) = ar^2 + br - \frac{c}{r}, \tag{1}$$

where a, b and c are constant potential parameters and will be fixed later on.

Formalism

In the present work, the Nikiforov-Uvarov Functional analysis (NUFA) method [3] is used to calculate the energy eigenvalues and corresponding eigen function for CPP. The N-dimensional radial SE for CPP is written as

$$\frac{d^2R(r)}{dr^2}+\frac{2\mu}{\hbar^2}\left[E-ar^2-br+\frac{c}{r}-\frac{\eta}{2\mu r^2}\right]R(r)=0, \tag{2}$$

where, $\eta = l(l+N-2)\hbar^2$. By using the Greene-Aldrich approximation scheme and the NUFA methodology [3], the final energy eigenvalue equation is given as

$$E=\frac{-\hbar^2\alpha^2}{2\mu}\left[\frac{-(\nu+n)^2+\tau_1-\tau_3}{2(\nu+n)}\right]^2+\frac{\hbar^2\alpha^2\tau_3}{2\mu}. \tag{3}$$

where

$$\tau_1 = \frac{12\mu a}{\hbar^2\alpha^4}+\frac{6\mu b}{\hbar^2\alpha^3}, \tag{4}$$

$$\tau_3 = \frac{2\mu a}{\hbar^2\alpha^4}+\frac{2\mu b}{\hbar^2\alpha^3}-\frac{2\mu c}{\hbar^2\alpha}+l(l+N-2), \tag{5}$$

and

$$\nu = \frac{1}{2}+\frac{1}{2}\sqrt{1+4(\tau_1-\tau_2+\tau_3)}. \tag{6}$$

and the corresponding eigen function is written as

$$R_{nl}(r) = N\left(e^{-\alpha r}\right)^{\sqrt{\varepsilon+\tau_3}}\left(1-e^{-\alpha r}\right)^{\frac{1}{2}+\frac{1}{2}\sqrt{1+4(\tau_4)}} \times {}_2\mathrm{F}_1(a_2,b_2,c_2;e^{-\alpha r}).$$

where N is normalization constant and ${}_2\mathrm{F}_1(a_2,b_2,c_2;e^{-\alpha r})$ is hypergeometric function. Here, the parameters a_2, b_2, c_2 and τ_4 are defined as

$$\begin{aligned} a_2 &= \left(\lambda+\nu+\sqrt{\xi_1}\right), \\ b_2 &= \left(\lambda+\nu-\sqrt{\xi_1}\right), \\ c_2 &= 1+2\lambda \ , \\ \tau_4 &= \tau_1-\tau_2+\tau_3. \end{aligned} \tag{7}$$

*Electronic address: fchand@kuk.ac.in

Mass Spectra of Mesons

To calculate the mass spectra of charmonium ($c\bar{c}$) and bottomonium ($b\bar{b}$) mesons, the following relation [4] is used

$$M = 2m + E_{nl}. \qquad (8)$$

Here we have used m_c=1.317 GeV and m_b=4.584 GeV.

Table 1: Mass spectra of Charmonium ($c\bar{c}$) in GeV. (a=0.0297 GeV^3, b=0.7609 GeV^2, c=60.586 and α=0.0959)

State	Present work	[1]	[5]	[6]
1S	3.091	3.094	3.097	3.097
2S	3.627	3.681	3.673	3.679
3S	4.121	4.129	4.022	4.078
4S	4.573	4.514	4.273	4.412
5S	4.988	4.863	4.463	4.711
6S	5.368	5.185	4.608	-
1P	3.123	3.468	3.510	3.516
2P	3.658	3.938	3.901	3.937
3P	4.150	4.338	4.178	4.284
4P	4.602	4.696	-	-
5P	5.016	5.026	-	-
1D	3.187	3.775	3.787	3.787
2D	3.819	4.188	4.089	4.144
3D	4.209	4.555	4.137	4.456
4D	4.658	4.891	-	-

Table 2: Mass spectra of bottomonium ($b\bar{b}$) in GeV. (a=0.0297 GeV^3, b=1.378 GeV^2, c=10.473 and α=0.281)

State	Present Work	[1]	[7]	[8]
1S	9.460	9.463	9.460	9.465
2S	9.981	9.979	10.016	10.003
3S	10.401	10.359	10.351	10.354
4S	10.736	10.683	10.611	10.635
5S	10.998	10.975	10.831	10.878
6S	11.196	11.243	11.023	11.102
1P	9.544	9.819	9.897	9.876
2P	10.056	10.217	10.251	10.246
3P	10.468	10.553	10.524	10.538
4P	10.796	10.853	10.753	10.788
5P	11.052	11.127	10.951	11.014
1D	9.808	10.074	10.145	10.138
2D	10.202	10.423	10.432	10.441
3D	10.599	10.731	10.670	10.698
4D	10.915	11.013	10.877	10.928

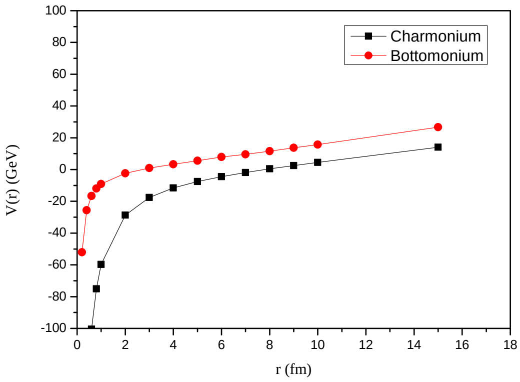

FIG. 1: Variation of the Coulomb perturbed potential with r.

Conclusion

Here in the present work, we have calculate the mass spectra of $c\bar{c}$ and $b\bar{b}$ mesons by CPP invoking NUFA method. The results presented in tables (1 & 2) are in excellent agreement with previous results obtained via quantum chromodynamics (QCD) formalism.

Acknowledgment

One of the authors, Vinod Kumar acknowledges the University Grant Commission (UGC), New Delhi (India) for providing financial support through NFSCS-UGC JRF Fellowship.

References

[1] N. R. Soni *et al.*, The European Physical Journal C **78**, 592 (2018).

[2] R. Kumar and F. Chand Comm. Theor. Phys. **59**, 528 (2013).

[3] A.N. Ikot *et al.*, Few-Body Systems **62**, 1 (2021).

[4] R. Rani, S. Bhardwaj, and F. Chand Comm. Theor. Phys. **70**, 179 (2018).

[5] B. Q. Li and K.-T. Chao, Physical Review D **79**, 094004 (2009).

[6] W.J. Deng *et al.*, Physical Review D **95**, 034026 (2017).

[7] L. B. Qing and C. Kuang-Ta Comm. Theor. Physics **52**, 653 (2009).

[8] S. Godfrey and K. Moats Physical Review D **92**, 054034 (2015).