Masses of Bc Meson from the point of view of Martin potential

Rameshri V. Patel[1],* Manan Shah[1],†
Bhoomika Pandya[2], and P. C. Vinodkumar[2]‡
[1]*P. D. Patel Institute of Applied Sciences,*
Charotar University of Science and Technology,
CHARUSAT Campus, Changa 388 421, Anand, Gujarat, India and
[2]*Department of Physics, Sardar Patel University, Vallabh Vidyanagar-388120, INDIA*

Introduction

The new results of LHCb collaboration suggest the violation of the lepton flavour universality from the decays of beauty quark[1], so it is essential to understand the beauty quark to understand the lepton flavour universality. We have already studied Bottomonium in the framework of a semi-classical approach to understand the beauty quark[3]. In this paper, we study charmed beauty meson in the framework of Martin potential.

Methodology

The different spectroscopic properties like Spin-averaged masses and decay widths of Quarkonium states were studied in Ref.[3].Here, we are using the semi-classical approach[3],[4] and considering the Martin potential[2].

$$V(r) = \lambda r^{\nu} + V_0(l=0) + Bl(l+1)\langle r^{-2}\rangle. \quad (1)$$

The λ is potential strength and index $\nu = 0.1$.

The potential is considered the centrifugal contribution for non-zero orbital angular momentum states for Bc meson.

S-Waves

For S-wave kinds of states (i.e., $l = 0$) the third term in eqn.(1) will not contribute and for the first two terms, we've used the experimental spin averaged masses [5] of S-waves. We have used experimental pseudoscalar and vector masses to get the theoretical ground state. The best-fitted values of unknown parameters are $V_0 = -5.7273 GeV$ & $\lambda = 6.0342 GeV^{(\nu+1)}$. Now to predict the vector and pseudoscalar masses of S-states, we have used the Spin-Averaged mass(M_{SA}) eqn.(2) and Hyperfine mass splitting(ΔM) eqn.(3) from Ref.[9].

$$M_{SA} = m_c + m_b + V_0 + E_{nl} \quad (2)$$

$$\Delta M = \frac{A_{hyp}\left|\psi_{nl}(0)\right|^2}{m_{c(eff)} m_{b(eff)}} \quad (3)$$

Where, $m_{c(eff)}$ & $m_{b(eff)}$ are the effective masses corresponds to charm and beauty quark respectively, m_c & m_b are the masses of charm and beauty quark[5], E_{nl} is the bound energy found from the semi-classical approximation[4], $\left|\psi_{nl}(0)\right|^2$ is the ground state wavefunction found from the Feynman-Hellman theorem and semi-classical approximation[4], and the best fitted value of hyperfine parameter is $A_{hyp} = 10.3274$. The splitted S-wave masses along with the available experimental data and results of other approaches are mentioned in the **TABLE I**

TABLE I : S-Waves of B_c^+ in GeV

$n^{2S+1}S_J$	Our mass	[6]	[7]	[8]	Experiment
1^1S_0	6.2735	6.272	6.271	6.278	6.2749 [5]
1^3S_1	6.3325	6.333	6.338	6.331	-
2^1S_0	6.8506	6.842	6.855	6.863	6.8716 [5]
2^3S_1	6.8791	6.882	6.887	6.873	-
3^1S_0	7.1719	7.226	7.25	7.244	-
3^3S_1	7.1913	7.258	7.272	7.249	-
4^1S_0	7.3991	7.585	-	7.564	-
4^3S_1	7.4139	7.609	-	7.568	-
5^1S_0	7.5762	7.928	-	7.852	-
5^3S_1	7.5882	7.947	-	7.855	-

---

*Electronic address: `rameshri.patel1712@gmail.com`
†Electronic address: `mnshah09@gmail.com`
‡Electronic address: `pothodivinod@yahoo.com`

P & D -Waves

Now the third term in eqn.(1) starts showing its effects in the case of non-zero orbital angular momentum states (i.e., $l \neq 0$). The fitting of centrifugal parameter B using the theoretical mass of $1P$ state [6][7][8] produces the value $0.0107967 GeV^{-1}$.

Non-zero orbital angular momentum states shows splitting only due to the spin-orbit and the Tensor interactions while the effects of the spin-spin interactions(hyperfine splitting) are negligible. the masses of triplet states ($S = 1$) n^3P_0 or 0^{++} (scalar), n^3P_1 or 1^{++} (pseudovector) & n^3P_2 or 2^{++} (tensor) can be found by adding contributions of interactions to the M_{SA}. We have considered eqn.(4) & (5) the spin-orbit and tensor contributions from the Breit interaction terms [10].

$$H_T = \frac{1}{12 m_c m_b} S_{12} \left[\frac{1}{r} \frac{d}{dr} V_V(r) - \frac{d^2}{dr^2} V_V(r) \right] (4)$$

$$H_{LS} = \frac{1}{2 m_c m_b} \vec{L}.\vec{S} \left[3 \frac{d}{dr} V_V(r) - \frac{d}{dr} V_s(r) \right] \quad (5)$$

where we took $V_V(r) = V_S(r) = \frac{1}{2} V(r)$, $S_{12} = \frac{4}{(2l-1)(2l+3)} \left[\vec{S}^2 \vec{L}^2 - \frac{3}{2} \vec{L}.\vec{S} - 3(\vec{L}.\vec{S})^2 \right]$ and $\vec{L}.\vec{S} = \frac{1}{2}[J(J+1) - S(S+1) - L(L+1)]$. So

$$\langle H_T \rangle = \frac{\lambda \nu}{12 m_c m_b} S_{12} \langle r^{-2} \rangle, \quad (6)$$

$$\langle H_{LS} \rangle = \frac{\lambda \nu}{2 m_c m_b} \vec{L}.\vec{S} \langle r^{-2} \rangle \quad (7)$$

where $\langle r^{-2} \rangle$ can also be found using the Feynman-Hellman theorem. The non-zero orbital angular momentum states are not yet found experimentally, so we mentioned our predictions with other theoretical approaches along with the contributions of interactions in **TABLE II** & **TABLE III**.

TABLE II : P-Wave of B_c^+ in GeV

$n^{2S+1}P_J$	Our M_{SA}	H_T	H_{LS}	Our mass	[6]	[7]	[8]
1^1P_1	6.75	0	0	6.75	6.75	6.75	6.769
1^3P_0		-0.0182	-0.0545	6.6774	6.699	6.706	6.748
1^3P_1		0.0091	-0.0272	6.7318	6.743	6.741	6.767
1^3P_2		-0.00182	0.0272	6.7754	6.761	6.768	6.775
2^1P_1	7.1059	0	0	7.1059	7.1430	7.15	7.156
2^3P_0		-0.0107	-0.0320	7.0633	7.094	7.122	7.139
2^3P_1		0.0053	-0.0160	7.0953	7.134	7.145	7.155
2^3P_2		-0.0010	0.0160	7.1209	7.157	7.164	7.162
3^1P_1	7.3498	0	0	7.3498	7.51	-	7.479
3^3P_0		-0.0076	-0.0229	7.3191	7.474	-	7.463
3^3P_1		0.0038	-0.0115	7.3421	7.5	-	7.479
3^3P_2		-0.0008	0.0115	7.3604	7.524	-	7.485

TABLE III : D-Wave of B_c^+ in GeV

$n^{2S+1}D_J$	Our M_{SA}	H_T	H_{LS}	Our mass	[6]	[7]	[8]
1^1D_1	7.0093	0	0	7.0093	7.026	7.036	7.035
1^3D_0		-0.0040	-0.0362	6.9691	7.021	7.03	7.028
1^3D_1		0.0040	-0.0121	7.0013	7.025	7.041	7.025
1^3D_2		-0.0011	0.0241	7.0323	7.029	7.045	7.026
2^1D_1	7.2777	0	0	7.2777	7.40	-	7.37
2^3D_0		-0.0027	-0.0240	7.2510	7.392	-	7.365
2^3D_1		0.0027	-0.0080	7.2724	7.399	-	7.361
2^3D_2		-0.0008	0.0160	7.2929	7.405	-	7.363
3^1D_1	7.4791	0	0	7.4791	7.743	-	-
3^3D_0		-0.0020	-0.0181	7.4589	7.732	-	-
3^3D_1		0.0020	-0.0060	7.4751	7.741	-	-
3^3D_2		-0.0006	0.0121	7.4907	7.75	-	-

Result and discussion

The good agreement of the masses with the available experimental data [5] and other theoretical approaches [6][7][8] is the indication of success of Martin potential in explaining the spectroscopic properties of B_C^+ meson.

References

[1] R. Aaij *et al.* [LHCb], [arXiv:2103.11769 [hep-ex]].

[2] A. Martin, Physics Letters B **93B**, 338-342 (1980).

[3] Shah Manan, Parmar Arpit and Vinodkumar P.C., Phys. Rev. D **86**, 034015 (2012).

[4] C. Quigg and Jonathan L. Rosner., Physics Reports. **56**, 167-235 (1979).

[5] P.A. Zylaet al.(Particle Data Group), Prog. Theor. Exp. Phys. **083C01**,(2020).

[6] D.Ebert ,R.N. Faustov and V.O.Galkin, Eur. Phys. J. **71**,1825(2011)

[7] Godfrey Stephen, Phys.Rev.D **70**,054017 (2004).

[8] N.Devlani, V.Kher, and A.K.Rai, Eur.Phys.J.A **50**,154 (2014).

[9] N. Barik and S.N. Jena, Physics Letters B **97**,261-264 (1980).

[10] I. Haysak, Yu. Fekete, V. Morokhovych, S. Chalupka, and M. Salak, Czech. J. Phys. **55**, 541 (2005).