Instanton Contribution to Orbitally Excited States of Bottomonium Mass Spectra*

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Introduction

Our Model uses the NRQM formalism for the study of properties of bottomonium states using a Hamiltonian which has the heavy quark potential derived from the instanton vacuum depending on r, the inter quark distance. The heavy quark potential derived from the instanton ensemble rises linearly as the relative distance between the quark and anti quark increases and gets saturated. The bound state of a bottom quark b and its anti quark $\bar{b}$ known as bottomonium, plays an important role in the study of the strong interactions.

Theoretical Background

In a potential model approach the entire dynamics of quarks in a meson is governed by a Hamiltonian has kinetic energy term (K) and a potential energy (V), that is [2],

$$K = M + \frac{p^2}{2\mu}$$

Here p is the relative momentum, $\mu = \frac{m_Q m_{\bar{Q}}}{m_Q + m_{\bar{Q}}}$ is the reduced mass of the $Q\bar{Q}$ system, where m_Q and $m_{\bar{Q}}$ are the masses of the individual quark and anti quark respectively and M is the total mass of quark and antiquark.The potential energy V is the sum of the heavy-quark potential $V_{Q\bar{Q}}(\vec{r})$, confining potential $V_{conf}(\vec{r})$ and Coulomb potential $V_{coul}(\vec{r})$.

$$V(\vec{r}) = V_{Q\bar{Q}}(\vec{r}) + V_{coul}(\vec{r}) + V_{conf}(\vec{r})$$
$$V_{Q\bar{Q}}(\vec{r}) = V_C(\vec{r}) + V_{SD}(\vec{r}).$$

Here $V_C(\vec{r})$ and $V_{SD}(\vec{r})$ are central and spin dependent potentials due to instanton vacuum respectively [4].

$V_C(\vec{r})$ is given by the following expression

$$V_C(\vec{r}) \simeq \frac{4\pi\bar{\rho}^3}{\bar{R}^4 N_c}\left(1.345\frac{r^2}{\bar{\rho}^2} - 0.501\frac{r^4}{\bar{\rho}^4}\right) \quad (1)$$

Here, $\bar{\rho} = \frac{1}{3}$ fm the average size of the instanton, $\bar{R} = 1$ fm the average separation between instantons and number of colors N_C is 3.

The spin-spin interaction $V_{SS}(\vec{r})$, the spin-orbit coupling term $V_{LS}(\vec{r})$ and the tensor part $V_T(\vec{r})$ contribute to the spin dependent potential $V_{SD}(\vec{r})$;

$$V_{SD}(\vec{r}) = V_{SS}(\vec{r}) + V_{LS}(\vec{r}) + V_T(\vec{r})$$
$$V_{SS}(\vec{r}) = \frac{1}{3{m_Q}^2}\nabla^2 V_C(\vec{r}); \;\; V_{LS}(\vec{r}) = \frac{1}{2{m_Q}^2}\frac{1}{r}\frac{dV_C(\vec{r})}{dr};$$
$$V_T(\vec{r}) = \frac{1}{3{m_Q}^2}\left(\frac{1}{r}\frac{dV_C(\vec{r})}{dr} - \frac{d^2V_C(\vec{r})}{dr^2}\right).$$

The coulomb-like (perturbative) one gluon exchange part of the potential is given by

$$V_{coul}(\vec{r}) = \frac{-4\alpha_s}{3r} \quad (2)$$

with the strong coupling constant α_s and inter quark distance r. The confinement term represents the non perturbative effect of QCD which includes the spin-independent linear confinement term [39].

$$V_{conf}(\vec{r}) = -\left[\frac{3}{4}V_0 + \frac{3}{4}cr\right]F_1 \cdot F_2 \quad (3)$$

where c and V_0 are constants. F is related to the Gell-Mann matrix, $F_1 = \frac{\lambda_1}{2}$ and $F_2 = \frac{\lambda_2^*}{2}$ and $F_1 \cdot F_2 = \frac{-4}{3}$ for the mesons. In our work, we have used the three-dimensional harmonic oscillator wave function which has been extensively used in atomic and nuclear physics is used as the trial

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wave function for obtaining the $Q\bar{Q}$ mass spectrum.

$$\psi_{nlm}(r,\theta,\phi) = N\left(\frac{r}{b}\right)^l L_n^{l+1/2}(\frac{r}{b})exp\left(-\frac{r^2}{2b^2}\right)Y_{lm}(\theta,\phi)$$

where $|N|$ is the normalizing constant given by

$$|N|^2 = \frac{2\alpha^3 n!}{\sqrt{\pi}}\frac{2^{(2(n+l)+1)}}{(2n+2l+1)!}(n+l)! \quad (4)$$

and $L_n^{l+1/2}(x)$ are the associated Laguerre polynomials,

Results and Discussions

In this model of nonrelativistic dynamics, instanton induced potential and Coulomb like OGEP plays superior role. The instanton effects on masses of orbitally excited state of bottomonia are listed in the table 1.

Table 2 The contributions from instantons (M_I) to the bottomonium (MeV)

$n^{2S+1}L_J$	The Mass	M_{exp} MeV	M_I MeV
$1\,^3P_0$	9856	9859.44±0.42	9.51
$1\,^3P_1$	9903	9892.78±0.26.	9.21
$1\,^1P_1$	9898	9899±0.8	8.61
$1\,^3P_2$	9916	9912.21±0.26.	8.53
$2\,^3P_0$	10244	10232.5±0.4	14.13
$2\,^3P_1$	10275	10255.46±0.022	15.01
$2\,^1P_1$	10269	10259.80±1.2	13.13
$2\,^3P_2$	10287	10268.65±0.22	15.15
$3\,^3P_0$	10566	....	14.12
$3\,^3P_1$	10560	....	14.21
$3\,^1P_1$	10589	10513.4±0.7	19.21
$3\,^3P_2$	10599	10524.0±3.8	19.23
$4\,^3P_0$	10857	....	19.76
$4\,^3P_1$	10876	......	18.13
$4\,^1P_1$	10751	....	18.60
$4\,^3P_2$	10882	......	19.03

The spin dependent terms of instanton potential give the hyperfine splitting. The OGEP is required as it is consistent with the asymptotic freedom.

Table 1 Instanton effects on the hyperfine mass splitting (MeV)

Mass Splittings	Present Work	Exp
$\Delta M(1^3P_2 - 1^3P_1)$	15.48	19.43
$\Delta M(2^3P_1 - 2^3P_0)$	46.95	33.34
$\Delta M(2^3P_2 - 2^3P_1)$	11.38	13.19
$\Delta M(2^3P_1 - 2^3P_0)$	31.62	22.96
$\Delta M(3^3P_2 - 3^3P_1)$	9.97	10.6

Instantons were introduced in relation to the UA(1) problem and their role was pointed out by t'Hooft by deriving effective interactions by coupling of the instantons and light quarks, whose strength of interaction depends on the instanton density, which was estimated from the gluon condensate of the QCD vacuum. It was argued that the NRQM should include the instanton potential as a short-range non-perturbative gluon effect. Also, lattice QCD suggests that the QCD vacuum contains instantons and its density is consistent with the gluon condensate expected from QCD sum rules. Also, it is well known that chiral symmetry is dynamically broken by the instanton vacuum and massless quarks are transformed into constituent quarks, which acquire mass as a func tion of momentum. Hence a constituent quark model Hamiltonian should have both OGEP and instanton potential. The results showed that the instanton-induced potentials contribute significantly to the mass spectrum of heavy quark mesons.

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