Study of s-state Ξ-hypernuclei in the Hyperspherical Harmonics Expansion approach

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Introduction

Hypernuclei are the subatomic systems with a remarkable strangeness degree of freedom as compared with conventional nuclei. Such exotic nuclei are formed when one or more strange exotic hyperon(s) (like Λ, Σ, Ξ etc) are injected into an atomic nucleus so as to replace one or more nucleon(s) of the nucleus and the injected particles are coupled to the nuclear core[1]. Hypernuclei provide a unique source of information on hyperon-nucleon, hyperon-hyperon interactions and the behaviour of strange particles in baryonic matter which opened an avenue for many significant phenomenon in particle physics and nuclear-astrophysics[2].

Nakazawa et al., 2015[3] first reported the bound state of the p-state $\Xi^- - {}^{14}$N hypernuclei with a Ξ^- separation energy of 4.38±0.25 MeV. Recently, Yoshimoto et al., 2021[4] first identified the nuclear 1s state of the hypernucleus ${}^{15}_{\Xi}$C having B_{Ξ^-} of IRRAWADDY (6.27±0.27 MeV) and KINKA (8.0±0.77 MeV) events. Some theoretical investigations were done to estimate whether the Ξ^- occupies on 1s state or 1p state in KISO event, such as relativistic mean-field (RMF) model, Skyrme-Hartree-Fock (SHF) model[5] and quark mean-field (QMF) model[6].

The main objective of the present work is to study the ground state in 1s state of ${}^{15}_{\Xi^-}$C and we will predict ${}^{16}_{\Xi^-\Xi^-}$C hypernnuclei. We will calculate the one- and two-Ξ^- separation energies and some relevant geometrical observables of these systems. In our two-body model, we will first solve Schrödinger equation numerically to reproduce the single Ξ^- separation energy of ${}^{15}_{\Xi^-}$C hypernuclei by adjusting the parameter of the effective core-hyperon potential. We use the Ξ-nucleus potential together with ΞΞ potential to predict the ground state energy of the ${}^{16}_{\Xi^-\Xi^-}$C hypernuclei. For the three-body model calculation of double-Ξ system, we employ the hyperspherical harmonics expansion (HHE) method.

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Theoretical formalism

In our HHE approach[7], we label the relatively heavier core nucleus ^{14}C as the particle "i" and two valence Ξ-hyperons as particles "j" and "k" form the interacting pair are:

$$\left.\begin{array}{rcl} \vec{\xi_i} &=& a_{jk}(\vec{r_j} - \vec{r_k}) \\ \vec{\chi_i} &=& a_{(jk)i}\left(\vec{r_i} - \frac{m_j\vec{r_j}+m_k\vec{r_k}}{m_j+m_k}\right) \\ \vec{R} &=& \sum_{i=1}^{3} \frac{m_i\vec{r_i}}{M} \end{array}\right\} \quad (1)$$

where $\rho = \sqrt{\xi_i^2 + \chi_i^2}; \phi_i = \tan^{-1}(\chi_i/\xi_i)$. The motion of the three-body system in relative coordinates can be described by the Schrödinger's equation

$$\left[-\frac{\hbar^2}{2\mu}\left\{\frac{\partial^2}{\partial\rho^2} + \frac{5}{\rho}\frac{\partial}{\partial\rho} + \frac{\hat{K}^2(\Omega_i)}{\rho^2}\right\}\right]\psi(\rho,\Omega_i) + \left[V(\rho,\Omega_i) - E\right]\psi(\rho,\Omega_i) = 0 \quad (2)$$

where $V(\rho, \Omega_i)$ is the total interaction potential in the partition i and $\hat{\mathcal{K}}^2(\Omega_i)$ is the square of hyper angular momentum operator satisfying the eigenvalue equation.

Choice of potential

For the core-Ξ subsystem and ΞΞ pair, we used a two-range Gaussian Isle-type potential and Yukawa-type potential [8] of the form re-

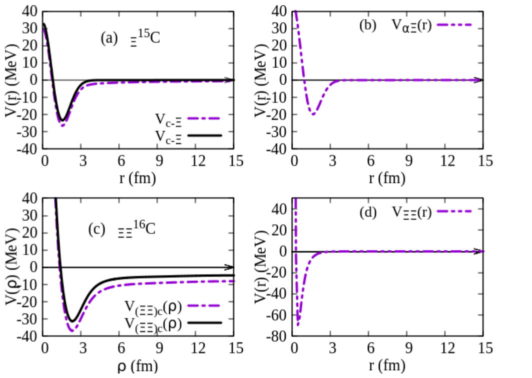

FIG. 1: Representative plot of effective core-Ξ potential components with radial distance are (a) with coulomb and without coulomb (solid line) interaction, (b) $\alpha - \Xi$ (d) ΞΞ pair and (c) for three-body effective potential with hyper-radial distance ρ.

spectively

$$V_{c-\Xi}(r) = 450.4\exp(-(r/1.269)^2) -404.9\exp(-(r/1.41)^2) \quad (3)$$

$$V_{\Xi\Xi}(r) = (-155.0\exp(-1.75r) +490.0\exp(-5.6r))/r \quad (4)$$

A Coulomb interaction is added for the charged pair of interacting particles

$$V_{ij}^{(C)}(r_{ij}) = \begin{cases} 1.44\left(\frac{Z_iZ_j}{2R_c}\right)\left(3 - \frac{r_{ij}^2}{R_c^2}\right) & ; \; r_{ij} \leq R_c \\ 1.44\left(\frac{Z_iZ_j}{r_{ij}}\right) & ; \; r_{ij} > R_c \end{cases} \quad (5)$$

Results and Discussions

In this work, we first investigate the 1s bound state of ${}^{15}_{\Xi^-}$C and predict double-Ξ hypernucleus ${}^{16}_{\Xi^-\Xi^-}$C using HHE method which is an essentially exact method, where calculations can be carried out up to any desired precision by gradually increasing the basis.

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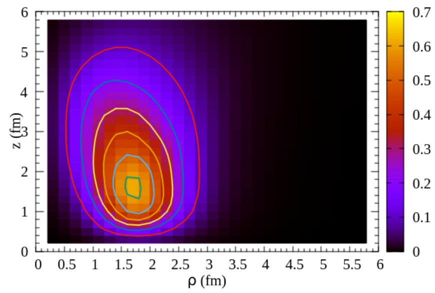

FIG. 2: Contour plot reflects probability density distribution of nucleons and Ξ^- hyperons in the ground state of ${}^{16}_{\Xi^-\Xi^-}$C.

TABLE I: (I) Parameters of the attractive component of the core-Ξ^- potential and B_{Ξ^-} separation energy of ${}^{15}_{\Xi^-}$C. (II) Double-Ξ^- separation energy and r.m.s raddi of ${}^{16}_{\Xi^-\Xi^-}$C. The bold face parameters obtained by switch off the coulomb interaction term. Energies and potentials are in MeV and raddi are in fm unit.

(I)	V_{att}	$V_{c-\Xi^-}$ *Min.*	B_{Ξ^-} Cal.	B_{Ξ^-} Expt.
${}^{15}_{\Xi^-}$C	-417.18	-26.3556	8.0012	8.0±0.77
		-23.2811	**5.1671**	
(II)	$B_{\Xi^-\Xi^-}$	R_A	$R_{c\Xi^-}$	$R_{\Xi^-\Xi^-}$
${}^{16}_{\Xi^-\Xi^-}$C	-20.543	2.581	2.198	2.716
	-15.280	**2.585**	**2.229**	**2.741**

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