Double charmonium production in the framework of Bethe-Salpeter equation

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Introduction

Double charmonium production in electron-positron annihilation at B-factories is one of the challenging areas of hadronic physics. It is seen that the perturbative QCD calculations fail at low energy range typically below the J/psi mass scale. However, at this scale we can employ the Bethe-Salpeter (BS) framework. In recent years, the quarkonium production has been studied in various process at B-factories whose measurements were made by Babar and Belle collaborations [1–2]. Among them, the study of charmonium production in e^+e^- annihilation is particularly interesting in testing the quarkonium production mechanisms at center of mass energies $s = 10.6GeV$. The investigation of double charmonium production is very important since these charmonium can be easily produced in experiments and hence their theoretical prediction can verify the discrepancy between different theoretical models and experimental data.

In these calculations, we make use of the Bethe-Salpeter Equation (BSE) approach [3-8], which is a conventional approach in dealing with relativistic bound state problems. Due to its firm base in quantum field theory and being a dynamical equation based approach, it provides a realistic description for analyzing hadrons as composite objects and can be applied to study not only the low energy hadronic processes but also the high energy production processes involving quarkonia as well.

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1. Cross section for double charmonium production

We start from the lowest-order Feynman diagrams for the process, $e^- + e^+ \rightarrow J/|Psi + \eta_c$ as given in Figure 1. There are four Feynman diagrams for the production of double charmonium, one of which is shown in Figure 1, while the other three can be obtained by reversing the arrows of the internal fermionic lines.

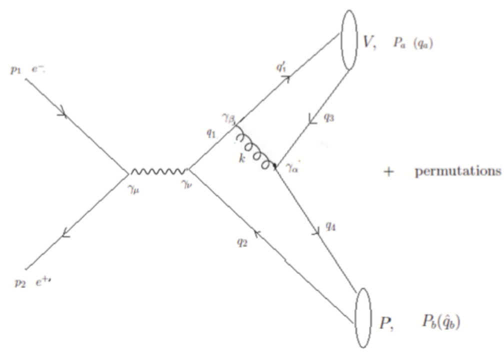

FIG. 1: Feynman diagrams for the production of double charmonium in e^+e^-- annihilation

The invariant amplitude M^1_{fi} for double charmonium production, corresponding to Fig.1, is given by the one-loop momentum integral as:

$$M^1_{fi} = \frac{2^7\pi^2\alpha_{em}\alpha_s}{3^2 s}\,[\overline{\nu}(p_2)\gamma_\mu u(p_1)]$$
$$\int \frac{d^4q_a}{(2\pi)^4}\int \frac{d^4q_b}{(2\pi)^4}$$
$$Tr[\overline{\Psi}(P_a, q_a)\gamma_\beta S_F(q_1)\gamma_\mu \overline{\Psi}(P_b, q_b)\gamma_\beta]\frac{1}{k^2}$$

where, s is the Mendelstam variable defined as, $s = -(p_1 + p_2)^2$, $\alpha_{em} = \frac{e^2}{4\pi}$ is the electromagnetic coupling constant and $\alpha_s = \frac{g_s^2}{4\pi}$ is the strong coupling strength, $\overline{\Psi}(P_a, q_a)$ and $\overline{\Psi}(P_b, q_b)$ are the adjoint BS wave function of vector and pseudoscalar charmonium respectively. From the figure, we can relate the momenta of the quark and anti-quark respectively as: $q_1' = \frac{1}{2}P_a + q_a$, $q_3 = \frac{1}{2}P_a - q_a$, $q_4 = \frac{1}{2}P_b + q_b$, and $q_2 = \frac{1}{2}P_b - q_b$ and the internal momenta for the propagators of gluon and quark are given by: $k = \frac{1}{2}(P_a + P_b) - q_a + q_b$, $q_1 = P_a + \frac{1}{2}P_b + q_b$. The total amplitude for the process $e^+ + e^- \to V + P$, can be obtained by summing over the amplitudes of all diagrams in Fig. 1. Then, the unpolarized total cross-section [3] is obtained by summing over various $V + P$, spin-states and averaging over those of the initial state, e^+e^-. After the integration of the phase space, the total cross-section for vector and pseudoscalar charmonium production is:

$$\sigma_{e^+e^-\to VP} = \frac{2^{27}\pi^3\alpha_{em}^2\alpha_s^2 m_c^2}{3^4 s^4}\left(1 - \frac{16m_c^2}{s}\right)^{\frac{3}{2}}$$
$$\left[\int \frac{d^3\widehat{q}_a}{(2\pi)^3} N_V \phi_V(\hat{q}_a)\right]^2 \left[\int \frac{d^3\widehat{q}_b}{(2\pi)^3} N_P \phi_P(\hat{q}_b)\right]^2$$

The theoretical values of total cross sections of pseudoscalar and vector charmonium production in our framework at $\sqrt{s} = 10.6GeV$. are listed in Table 2 [3].

TABLE I: Total cross sections (in fb) of pseudoscalar and vector charmonium production for ground and first excited states along with experimental data.

Process	σ(BSE=CIA)	σ[1]	σ[2]
$e^+e^- \to J/\psi\eta_c$	20.770	25.6±2.8	17.6±2.8
$e^+e^- \to \psi'\eta_c$	11.643	16.3±4.6	
$e^+e^- \to J/\psi\eta_c'$	11.177	16.5±3.0	16.4±3.7
$e^+e^- \to \psi'\eta_c'$	6.266	16.0±5.1	

This calculation involving production of double charmonia in electron-positron annihilation can be easily extended to studies on other processes (involving the exchange of a single virtual photon) observed at B-factories such as, $e^+e^- \to J/\Psi\chi_{c0}$ and $e^+e^- \to \gamma J/\Psi$. The amplitude M_{fi}^1 for the process, $e^+e^- \to J/\Psi\chi_{c0}$ can be expressed as,

$$M_{fi}^1 = \frac{2^{14}\pi^2\alpha_{em}\alpha_s m_c}{3^2 s^3}\left[\overline{\nu}(p_2)\gamma_\mu u(p_1)\right]$$
$$\epsilon_{\mu\nu\rho\sigma}\varepsilon_\mu P_a^\rho P_b^\sigma \int \frac{d^3\widehat{q}_a}{(2\pi)^3} N_S \phi_S(\hat{q}_a)$$
$$\int \frac{d^3\widehat{q}_b}{(2\pi)^3} N_P \phi_P(\hat{q}_b)$$

where the radial wave function $\phi_S(\hat{q})(1S)$, for scalar, and $\phi_S(\hat{q})(1S)$ pseudoscalar quarkonia obtained by analytic solutions [5] of their mass spectral equation are:

$$\phi_S(1P, \hat{q}) = \sqrt{\frac{2}{3}}\frac{1}{\pi^{3/4}}\frac{1}{\beta_S^{5/2}}\hat{q}e^{-\frac{\hat{q}^2}{2\beta_S^2}}$$
$$\phi_P(1S, \hat{q}) = \frac{1}{\pi^{3/4}\beta_P^{3/2}}e^{-\frac{\hat{q}^2}{2\beta_P^2}}$$

We intend to study the cross section for the above process. Among other similar works, in [6], the authors study the production ratio of neutral to charged kaon pair in e^+e^- annihilation below the J/ψ mass by employing the two leading Dirac structures in hadronic wave functions as per the power counting rule [7] we had proposed some time ago. We wish to extend this study to calculation of cross section of these processes with the involvement of all the possible Dirac structures.

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