Phase-Equivalent Potential Construction by SUSY Transformations

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INTRODUCTION

It is known that by the judicious use of the algebra of supersymmetry, a potential can be constructed with bound states at arbitrary energies. The SUSY transformation, a mathematical transformation which falls under SUSY algebra, is used to generate phase equivalent potentials with some desired alterations. There are 4 different types of SUSY transformations, using which, a supersymmetric partner to the Hamiltonian of a given radial Schrödinger equation can be formed. Using three pairs of successive SUSY transformation (T1T2, T1T3, and T1T4) on Manning-Rosen potential [2], three new phase-equivalent potentials namely V12; V13; and V14 are successfully constructed. As a model calculation we shall apply them to n-p, p-p and α-p systems.

Construction of Phase equivalent potential:

The three parameter short ranged Manning-Rosen potential [2]

$$V_m = b^{-2}\left[\frac{\alpha(\alpha-1)}{\left(1-e^{-r/b}\right)^2}e^{-2r/b} + \frac{l(l+1)}{\left(1-e^{-r/b}\right)^2}e^{-2r/b} - A\frac{e^{-r/b}}{\left(1-e^{-r/b}\right)}\right] \quad (1)$$

which is generally used for the study of molecular systems, have been parameterized to fit nuclear scattering phase parameters. The Jost solution for the potential under consideration reads

$$f_l(kr) = \left(1-e^{-r/b}\right)^{1+p} e^{-wr} \quad (2)$$

Under the pairs of SUSY transformations, three new potentials are obtained as

$$\mathrm{V12(r)} = \mathrm{Vm(r)} - \frac{\mathrm{d}^2}{\mathrm{dr}^2}\ln\left(1+\lambda\int_0^{\mathrm{r}} f_l^2(\mathrm{x})\mathrm{dx}\right) \quad (3)$$

$$V13(r) = \mathrm{Vm(r)} - \frac{\mathrm{d}^2}{\mathrm{dr}^2}\ln\left(\int_0^{\mathrm{r}} f_l^2(\mathrm{x})\mathrm{dx}\right) \quad (4)$$

and

$$\mathrm{V14(r)} = \mathrm{Vm(r)} - \frac{\mathrm{d}^2}{\mathrm{dr}^2}\ln\left(\int_{\mathrm{r}}^{\infty} f_l^2(\mathrm{x})\mathrm{dx}\right). \quad (5)$$

Where b, A and α are three adjustable parameters of Manning-Rosen potential. The quantity λ is an adjustable parameter for transformation operations, $w = -ikr$; and $p = \frac{1}{2}\left[1 \pm \sqrt{1+4\{\alpha(\alpha-1)+l(l+1)\}}\right] - 1.$

For p-p and α-p systems the electromagnetic interaction part is provided by addition of atomic Hulthén potential which acts as a Screened Coulomb potential:

$$V_H(r) = \frac{V_0\exp(-\mathrm{r}/\mathrm{a})}{(1-\exp(-\mathrm{r}/\mathrm{a}))}, \mathrm{a} > 0\,, \quad (6)$$

where V_0 is strength and a is screening radius.

We generate phase shifts for all the aforementioned states via phase function method, and compare the findings with standard data.

Results and Discussion:

For n-p, p-p system we have used $\frac{\hbar^2}{2m} = 41.47\ Mev\ fm^2$ and for α-p systems $\frac{\hbar^2}{2m} = 25.92\ Mev\ fm^2$

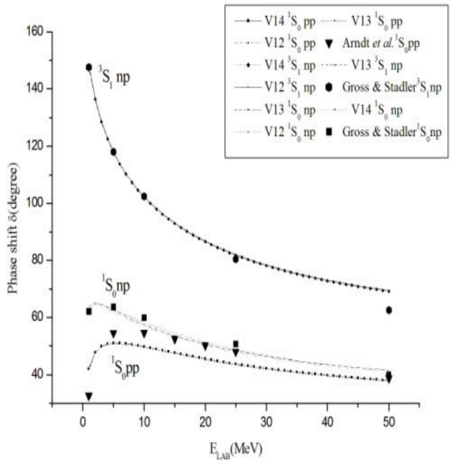

Fig 1. Graph for n-p; p-p Systems for V14, V13 and V12 potentials with reference data ($^1S_0$ & $^3S_1$ states).

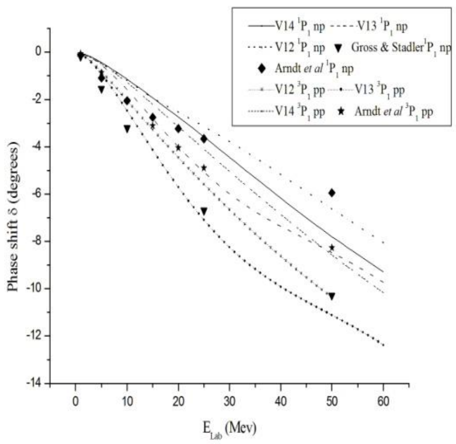

Fig 2. Graph for $^1P_1$ n-p & $^3P_1$ p-p phase shifts.

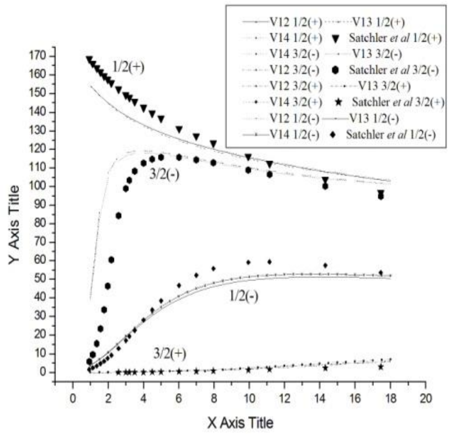

Fig 3. Graph for 1/2(+), 1/2(-), 3/2(+) and 3/2(-) α-p phase-shifts.

The $^1S_0$ & $^3S_1$ states for n-p system and $^1S_0$ state for p-p system are in good agreement with standard data [3] & [4] up to 50 MeV**.** For α-p system 1/2(+), 1/2(-), 3/2(+) and 3/2(-) States are in good agreement with experimental data [5]. The phase parameters for $^1P_1$ n-p and $^3P_1$ p-p states follow similar trends with reference data [3] and [4] with slight deviations in their numerical values.

Thus, we conclude that the SUSY transformations work to some good extent for generation of various phase equivalent potentials.

Reference :

1. C. V. Sukumar, J. Phys. **A:** Math. Gen. **21** L455-458 (1988)**.**
2. M. F. Manning and N. Rosen, Phys. Rev. **44** 953 (1933).
3. F. Gross and A. Stadler, *Phys. Rev*. C **78** 014005 (2008).
4. R. A, Arndt, L. D. Roper, R. A. Bryan, R. B. Clark, B. J. Ver West and P. Signell, *Phys. Rev.* D **28** 97 (1983).
5. G. R. Satchelar, L. W. Owen, A. J. Elwin, G. L. Morgan and R. L. Walter, *Nucl. Phys. A* **112** 1 **(**1968**).**