INTRODUCTION

It is known that by the judicious use of the algebra of supersymmetry, a potential can be constructed with bound states at arbitrary energies. The SUSY transformation, a mathematical transformation which falls under SUSY algebra, is used to generate phase equivalent potentials with some desired alterations. There are 4 different types of SUSY transformations, using which, a supersymmetric partner to the Hamiltonian of a given radial Schrödinger equation can be formed. Using three pairs of successive SUSY transformation (T1T2, T1T3, and T1T4) on Manning-Rosen potential [2], three new phase-equivalent potentials namely V12; V13; and V14 are successfully constructed. As a model calculation we shall apply them to n-p, p-p and α-p systems.

Construction of Phase equivalent potential:

The three parameter short ranged Manning-Rosen potential [2]

$$V_0 = b^2 \left[ \frac{\alpha (\alpha - 1)}{(1 - e^{-\alpha \Gamma})^2} e^{-\alpha \Gamma} + \frac{l(l + 1)}{(1 - e^{-\alpha \Gamma})^2} e^{-2 \alpha \Gamma} - \frac{A}{(1 - e^{-\alpha \Gamma})} \right]$$

(1)

which is generally used for the study of molecular systems, have been parameterized to fit nuclear scattering phase parameters. The Jost solution for the potential under consideration reads

$$f_i(kr) = (1 - e^{-\gamma/k})^{1+p} e^{-\gamma r}$$

(2)

Under the pairs of SUSY transformations, three new potentials are obtained as

$$V12(r) = V_0 + \frac{d^2}{dr^2} \ln \left( 1 + \lambda \int_0^r f_i^2(x) dx \right)$$

(3)

$$V13(r) = V_0 - \frac{d^2}{dr^2} \ln \left( \int_0^r f_i^2(x) dx \right)$$

(4)

and

$$V14(r) = V_0 - \frac{d^2}{dr^2} \ln \left( \int_0^r f_i^2(x) dx \right).$$

(5)

Where b, A and α are three adjustable parameters of Manning-Rosen potential. The quantity λ is an adjustable parameter for transformation operations, w = −ikr; and p = $$\frac{1}{2} \left[ 1 \pm \sqrt{1 + 4(\alpha (\alpha - 1) + l(l + 1))} \right] - 1$$.

For p-p and α-p systems the electromagnetic interaction part is provided by addition of atomic Hulthén potential which acts as a Screened Coulomb potential:

$$V_H(r) = \frac{V_0 \exp(-\gamma/k)}{(1 - \exp(-\gamma/k))}, \ a > 0,$$

(6)

where V0 is strength and a is screening radius.

We generate phase shifts for all the aforementioned states via phase function method, and compare the findings with standard data.
Results and Discussion:

For n-p, p-p system we have used
\[ \frac{h^2}{2m} = 41.47 \text{ MeV fm}^2 \]
and for α-p systems
\[ \frac{h^2}{2m} = 25.92 \text{ MeV fm}^2 \]

Fig 1. Graph for n-p; p-p Systems for V14, V13 and V12 potentials with reference data (1S0 & 3S1 states).

Fig 2. Graph for 1P1 n-p & 3P1 p-p phase shifts.

Fig 3. Graph for 1/2(+), 1/2(-), 3/2(+) and 3/2(-) α-p phase-shifts.

The 1S0 & 3S1 states for n-p system and 1S0 state for p-p system are in good agreement with standard data [3] & [4] up to 50 MeV. For α-p system 1/2(+), 1/2(-), 3/2(+) and 3/2(-) States are in good agreement with experimental data [5]. The phase parameters for 1P1 n-p and 3P1 p-p states follow similar trends with reference data [3] and [4] with slight deviations in their numerical values.

Thus, we conclude that the SUSY transformations work to some good extent for generation of various phase equivalent potentials.

Reference: