M1 and E1 transitions involving vector, pseudoscalar and scalar quarkonia in framework of Bethe-Salpeter equation

Shashank Bhatnagar[1] and Eshete Gebrehana[2*]
[1]*Department of Physics, Chandigarh University, Mohali-140413, India and*
[2]*Deptartment of Physics, Woldia university, Woldia, Ethiopia*

Introduction

The radiative transitions of heavy quarkonia are of considerable experimental and theoretical interest, and provide an insight into the dynamics of quarkonium. The radiative transitions [1] between 0^{-+} (pseudoscalar), and 1^{--} (vector) mesons (for instance, $J/\Psi(nS) \to \eta_c(n'S) + \gamma$), which proceeds through the emission of photon is characterized by $\Delta L = 0$, there is change in C-parity between the initial and final hadron states, though the total C-parity is conserved. These are the magnetic dipole transitions, $M1$, which are sensitive to relativistic effects. The E1 transitions are characterized by $|\triangle L| = 1$, and there is change in parity between the initial and final hadronic states, for instance, $\Psi(2S) \to \chi_c(1P) + \gamma$ or $\chi_c(1P) \to J/\Psi(1S) + \gamma$. In both M1 and E1 transitions, C-parity is conserved. Electric dipole transitions are much stronger than magnetic dipole transitions, and involve transitions between excited states.

In this work we focus on the radiative decays of the charmed and bottom vector mesons through the processes, $V \to P\gamma$, $V \to S\gamma$, and $S \to V\gamma$, where, V, P, S refer to vector, pseudoscalar and scalar quarkonia. In this work we study the radiative decays [2-6] of heavy-light quarkonia through M1 and E1 transitions that involve quark-triangle diagrams with two hadron vertices. We have expressed the transition amplitude M_{fi} as a linear superposition of terms involving all possible combinations of $++$, and $--$ components of Salpeter wave functions of final and initial hadron, with coefficients being related to results of pole integrations over complex σ-plane [2]. We have calculated the above M1 and E1 transitions. We have used algebraic forms of Salpeter wave functions [3-5] obtained through analytic solutions of mass spectral equations for ground and excited states of $0^{++}, 1^{--}$, and 0^{-+} heavy-light quarkonia [3–6] in approximate harmonic oscillator basis to calculate their decay widths. The input parameters used by us were obtained by fitting to their mass spectra. We have compared our results with experimental data and other models, and found reasonable agreements.

Radiative decays of heavy-light quarkonia through $V \to P\gamma$

To apply the framework of BSE to study radiative decays, $V- > P\gamma$, we have to remember that there are two Lorents frames, one the rest frame of the initial meson, and the other, the rest frame of final meson. To calculate further, we first write relationship between the momentum variables of the initial and final meson. Here, P, and q are the total momentum and the internal momentum of initial hadron, while P', and q' are the corresponding variables of the final hadron, and let k, and $\epsilon^{\lambda'}$ be momentum and polarization vectors of emitted photon, while ϵ^{λ} be the polarization vector of initial meson. Thus if $p_{1,2}$, and $p'_{1,2}$ are the momenta of the two quarks in initial and final hadron respectively.

We decompose the internal momentum q of the initial hadron into two components, $q = (\hat{q}, iM\sigma)$, where $\hat{q}_\mu$ is the component of internal momentum transverse to P such that $\hat{q}.P = 0$, while σ is the longitudinal component in the direction of P. Since we study the process in the frame of initial hadron, we decompose the internal momentum, q' of final

---

*Electronic address: `shashank_bhatnagar@yahoo.com`

meson into two components $q' = (\hat{q}', iM\sigma')$, with $\hat{q}' = q' - \sigma' P$ transverse to initial hadron momentum, P, and $\sigma' = \frac{q'.P}{P^2}$, longitudinal to P. Thus, $P.\hat{q}' = 0$. We now first try to find the relationship between the transverse components of internal momenta of the two hadrons, $\hat{q}$, and $\hat{q}'$. In the rest frame of initial meson, we can relate the components of internal momenta of the two hadrons as, $\hat{q}' = \hat{q} + \hat{m}_2\hat{P}'$, and $\sigma' = \sigma + \alpha$, where, $\alpha = \hat{m}_2\frac{M'^2 - M^2}{2M^2}$. The EM transition amplitude of the process is

$$M_{fi} = -i\int \frac{d^4q}{(2\pi)^4} Tr[e_q\overline{\Psi}_P(P', q')\not{\epsilon}^{'}\Psi_V(P, q)$$

$$S_F^{-1}(-p_2) + e_{\overline{Q}}\overline{\Psi}_P(P', q')S_F^{-1}(p_1)\Psi_V(P, q)\not{\epsilon}^{'}],$$

where M_{fi} is written in the rest frame of the initial hadron. Here, the first term corresponds to the first diagram, where the photon is emitted from the quark (q), while the second term corresponds to the second diagram where the photon is emitted from the antiquark $(\overline{Q})$ in vector meson. The transition amplitude, M_{fi} for the process, $V \rightarrow P\gamma$ in terms of the transition form factor, F_{VP} is expressed as [2],

$$M_{fi} = F_{VP}\ \epsilon_{\mu\nu\alpha\beta}P_\mu\epsilon_\nu^{\lambda'}\epsilon_\alpha^\lambda P'_\beta,$$

where the antisymmetric tensor, $\epsilon_{\mu\nu\alpha\beta}$ ensures its gauge invariance. The decay width, Γ in turn can be expressed as [2], $\Gamma = \frac{\alpha_{e.m.}}{3}|F_{VP}|^2\omega_k^3$, where, ω_k is the kinematically allowed energy of the emitted photon. Decay widths for $V \rightarrow P\gamma$ calculated in [2] along with experimental data are listed in Table 1

Radiative decays of heavy-light quarkonia through $V \rightarrow S\gamma$

The amplitude, M_{fi} for this process can be finally expressed as [2],

$$M_{fi} = S_1(\epsilon^{\lambda'}.\epsilon^\lambda) + S_2\beta(\epsilon^{\lambda'}.P)(\epsilon^\lambda.P'),$$

$$S_1 = -ieN_SN_V\frac{1}{M^2}\int\frac{d^3\hat{q}}{(2\pi)^3}\frac{\phi_S(\hat{q}')\phi_V(\hat{q})}{16\omega_1\omega_2\omega'_1\omega'_2}\Theta_1,$$

$$S_2 = -ieN_SN_V\frac{1}{M^2}\int\frac{d^3\hat{q}}{(2\pi)^3}\frac{\phi_S(\hat{q}')\phi_V(\hat{q})}{16\omega_1\omega_2\omega'_1\omega'_2}\Theta_2,$$

where, S_1, and S_2 are the form factors, with detailed expressions in [2]. We give the results of M1 transition, $V \rightarrow P\gamma$, and E1 transition, $V \rightarrow S\gamma$ obtained in BSE framework in Table 1 along with experimental data[1]. The aim of doing this study was to mainly test our analytic forms of wave functions obtained as solutions of mass spectral equations in an approximate harmonic oscillator basis obtained analytically from 4 × 4 BSE as a starting point, that has so far given good predictions [17–19] not only of the mass spectrum of heavy-light quarkonia, but also their transitions. The present work would in turn lead to the validation of our approach, which provides a much deeper insight than the purely numerical calculations in 4 × 4 BSE approach that are prevalent in the literature.

TABLE I: Radiative decay widths of heavy-light mesons (in Kev) for M1 and E1 transitions calculated in BSE, along with experimental data

	BSE-CIA[2]	Expt.[1]
$\Gamma_{J/\psi(1S_1)\rightarrow\eta_c(1S_0)\gamma}$	1.7035	1.5793±0.0112
$\Gamma_{\psi(2S_1)\rightarrow\eta_c(2S_0)\gamma}$	0.1820	0.2002±0.008
$\Gamma_{D^*(1S_1)\rightarrow D(1S_0)\gamma}$	1.2843	1.3344±0.0072
$\Gamma_{D^*(2S_1)\rightarrow D(2S_0)\gamma}$	0.1381	
$\Gamma_{B_c^*(1S_1)\rightarrow B_c(1S_0)\gamma}$	0.0664	
$\Gamma_{\psi(2S_1)\rightarrow\chi_{c0}(1P_0)\gamma}$	33.3985	28.5714±0.0432
$\Gamma_{\psi(3S_1)\rightarrow\chi_{c0}(2P_0)\gamma}$	61.6924	
$\Gamma_{\psi(3S_1)\rightarrow\chi_{c0}(1P_0)\gamma}$	1.815	
$\Gamma_{D^*(2S_1)\rightarrow D(1P_0)\gamma}$	1.0214	
$\Gamma_{B_c^*(2S_1)\rightarrow B_c(1P_0)\gamma}$	9.6489	

References

[1] P.A.Zyla et al., (Particle Data Group), Prog. Theo. Expt. Phys. 2020, 083C01 (2020).

[2] S.Bhatnagar, E.Gebrehana, Phys. Rev. D102, 094024 (2020).

[3] E.Gebrehana, S.Bhatnagar, H.Negash, Phys. Rev. D 100, 054034 (2019).

[4] H.Negash, S.Bhatnagar, Intl. J. Mod. Phys. E25, 1650059 (2016)

[5] S.Bhatnagar, L. Alemu, Phys. Rev. D97, 034021 (2018).

[6] V.Guleria, E.Gebrehana, S.Bhatnagar, arxiv:2107.02143[hep-ph].