To study the mass of Λ_c and Λ_b baryon

Kaushal Thakkar[1]* and Ajay Majethiya[2]
[1] *Department of Physics, Government College Daman, Daman-396210, Affiliated to VNSGU, Surat 395007, India*
and
[2] *V.S. Patel College of Arts & Science, Bilimora Affiliated to VNSGU, Surat 395007, Gujarat, India.*

Introduction

In beginning of the twenty first century significant progress has been achieved to study the baryons containing heavy quarks (c, b) by the world wide experimental facilities at Belle, BaBar, CLEO, CDF, SELEX, ALICE, LHC etc., as well as the various theoretical approaches. All these experimental measurements and theoretical calculations make the study of heavy baryons interesting.

Framework for Baryons

In this study, we have adopted Hypercentral Constituent Quark Model (HCQM) to study masses of heavy baryons (Λ_c and Λ_b). The relevant degrees of freedom for the relative motion of the three constituent quarks are provided by the relative Jacobi coordinates $\vec{\rho}$ and $\vec{\lambda}$ which are given by [1] as

$$\vec{\rho} = \frac{1}{\sqrt{2}}(\vec{r_1} - \vec{r_2} \quad (1)$$

$$\vec{\lambda} = \frac{m_1\vec{r_1} + m_2\vec{r_2} - (m_1 + m_2)\vec{r_3}}{\sqrt{m_1^2 + m_2^2 + (m_1 + m_2)^2}} \quad (2)$$

The respective reduced masses are given by

$$m_\rho = \frac{2m_1m_2}{m_1 + m_2} \quad (3)$$

$$m_\lambda = \frac{2m_3(m_1^2 + m_2^2 + m_1m_2)}{(m_1 + m_2)(m_1 + m_2 + m_3)} \quad (4)$$

Here, m_1, m_2 and m_3 are the constituent quark masses. The angles of the Hyperspherical coordinates are given by $\Omega_\rho = (\theta_\rho, \phi_\rho)$ and $\Omega_\lambda = (\theta_\lambda, \phi_\lambda)$. We define hyper radius x and hyper angle ξ by,

$$x = \sqrt{\rho^2 + \lambda^2} \ and \ \xi = arctan\left(\frac{\rho}{\lambda}\right) \quad (5)$$

In the center of mass frame ($R_{c.m.} = 0$), the kinetic energy operator can be written as

$$\frac{P_x^2}{2m} = -\frac{\hbar^2}{2m}(\triangle_\rho + \triangle_\lambda) = -\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2} + \frac{5}{x}\frac{\partial}{\partial x} + \frac{L^2(\Omega)}{x^2}\right) \quad (6)$$

where $m=\frac{2m_\rho m_\lambda}{m_\rho+m_\lambda}$ is the reduced mass and $L^2(\Omega) = L^2\ (\Omega_\rho,\Omega_\lambda,\xi)$ is the quadratic Casimir operator of the six-dimensional rotational group O(6) and its eigenfunctions are the hyperspherical harmonics, $Y_{[\gamma]l_\rho l_\lambda}(\Omega_\rho,\Omega_\lambda,\xi)$ satisfying the eigenvalue relation, $L^2Y_{[\gamma]l_\rho l_\lambda}(\Omega_\rho,\Omega_\lambda,\xi)$=-$\gamma(\gamma + 4)Y_{[\gamma]l_\rho l_\lambda}(\Omega_\rho, \Omega_\lambda, \xi)$. Here, l_ρ and l_λ are the angular momenta associated with the $\vec{\rho}$ and $\vec{\lambda}$ variables respectively and γ is the hyper angular momentum quantum number.

The model Hamiltonian for baryons in the HCQM is then expressed as

$$H = \frac{P_x^2}{2m} + V(x) \quad (7)$$

The six-dimensional hyperradial *Schrödinger* equation which corresponds to the above Hamiltonian can be written as

$$\left[\frac{d^2}{dx^2} + \frac{5}{x}\frac{d}{dx} - \frac{\gamma(\gamma + 4)}{x^2}\right]\psi_{\nu\gamma}(x) = -2m\left[E - V(x)\right]\psi_{\nu\gamma}(x) \quad (8)$$

*Electronic address: kaushal2physics@gmail.com

where $\psi_{\nu\gamma}(x)$ is the hyper-radial wave function. For the present study, we consider the hypercentral potential $V(x)$ as the hyper Coulomb plus linear potential which is given as

$$V(x) = \frac{\tau}{x} + \beta x + V_0 \tag{9}$$

Here, the hyper-Coulomb strength is $\tau = -\frac{2}{3}\alpha_s$, where $\frac{2}{3}$ is the color factor for the baryon. The term β corresponds to the string tension of the confinement. We fix the model parameters β and V_0 to get the experimental ground state mass of Λ_b baryon. The parameter α_s corresponds to the strong running coupling constant, which is written as

$$\alpha_s = \frac{\alpha_s(\mu_0)}{1 + \left(\frac{33-2n_f}{12\pi}\right)\alpha_s(\mu_0) ln\left(\frac{m_1+m_2+m_3}{\mu_0}\right)} \tag{10}$$

In the above equation, the value of α_s at $\mu_0 = 1$ GeV is considered 0.6 as shown in Table I. The six-dimensional hyperradial Schrdinger equation described by equation (8) has been solved in the variational scheme with the hyper-Coloumb trial radial wave function which is given by

$$\psi_{\nu\gamma} = \left[\frac{(\nu-\gamma)!(2g)^6}{(2\nu+5)(\nu+\gamma+4)!}\right]^{\frac{1}{2}} (2gx)^{\gamma} \times e^{-gx} L^{2\gamma+4}_{\nu-\gamma}(2gx) \tag{11}$$

The wave function parameter g and hence the energy eigenvalue are obtained by applying virial theorem. The baryon masses are determined by the sum of the model quark masses, kinetic energy and potential energy as

$$M_B = \sum_i m_i + \langle H \rangle \tag{12}$$

Result and Discussions

We have chosen the quark mass parameters as $m_u = 0.33$ GeV, $m_d = 0.35$ GeV, $m_c = 1.55$ GeV and $m_b = 4.95$ GeV (See Table I) to calculate the masses of Λ_c and Λ_b baryons in the Hypercentral Constituent Quark Model (HCQM).

TABLE I: Quark mass parameters (in GeV) and constants used in the calculations.

m_u	m_d	m_c	m_b	n_f	α_s (μ_0=1 GeV)
0.330	0.350	1.55	4.95	4	0.6

The computed masses of Λ_c and Λ_b baryons are mentioned in Table II. The calculated mass of Λ_c baryon is 2.232 GeV and the mass of Λ_b baryon is 5.619 which is in good agreement with the experimental results and the other model predictions. Here, we can say that the HCQM gives plausible predictions for the ground state mass of Λ_c and Λ_b baryon.

TABLE II: Masses of Λ_c and Λ_b Baryons in GeV.

M_{Λ_c}	Reference	M_{Λ_b}	Reference
2.232	This work	5.619	This work
2.272	[2]	5.619	[7]
2.268	[3]	5.612	[8]
2.285	[4]	5.618	[4]
2.286	[5]	5.619	[5]
2.286	[6]	5.620	[6]
2.286	PDG [9]	5.619	PDG [9]

References

[1] K. Thakkar, Eur. Phys. J. C 80, 926 (2020).
[2] S. Miguraa et al., Eur. Phys. J. A 28, 41 (2006).
[3] W. Roberts, M. Pervin, Int. J. Mod. Phys. A 23, 2817 (2008).
[4] T. Yoshida, et al., Phys. Rev. D 92, 114029 (2015).
[5] B. Chen, et al., Eur. Phys. J. A 51, 82 (2015).
[6] D. Ebert, R.N. Faustov, V.O. Galkin, Phys. Rev. D 84, 014025 (2011).
[7] Ke-Wei Wei, et al., arXiv:1609.02512v1 [hep-ph].
[8] Y. Yamaguchi, et al., Phys. Rev. D 91, 034034 (2015).
[9] P.A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020) and 2021 update.