Effects of the BGK collision integral on the charge and heat transport in a strongly magnetized thermal QCD medium

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Introduction

In the extreme conditions of temperature and/or density hadrons get deconfined into a novel state of strongly interacting matter, commonly termed as Quark Gluon Plasma (QGP). A strong magnetic field of magnitude m_π^2 at RHIC and $15m_\pi^2$ at LHC is also produced at very early stages of the collisions due to the relative motion of the spectator quarks in noncentral events at URHICs. It has been found in some recent studies that the time scales between the formation of the locally equilibrated thermal QCD medium and the production of strong magnetic field, is almost similar due to faster thermalization. Thus, the charge and heat transport coefficients of the medium will also get influenced by this intense magnetic field. The transport coefficients can be calculated using the relativistic Boltzman transport equation (RBTE) and the complexities of the collision integral can be avoided by using the relaxation time approximation (RTA). This type of collision term has a flaw that particle number is not instantaneously conserved but only on the average over a cycle. This can be overcome by taking the Bhatnagar-Gross-Krook (BGK) type collision term. In order to calculate the electrical and thermal conductivities, we have followed the kinetic theory approach with BGK collision term in RBTE. We have incorporated the interaction among the parton through effective thermal masses obtained from the pole of the resummed quark propagator, calculated in the perturbative thermal QCD with a strong magnetic field in the background.

The RBTE in the presence of the strong magnetic field can be written as

$$p^\mu \frac{\partial f_i^B}{\partial x^\mu} + q_i\ F^{\rho\sigma} p_\sigma \frac{\partial f_i^B}{\partial p^\rho} = C[f] \qquad (1)$$

where $f_i^B = f_{\text{eq},i}^B + \delta f_i^B$, $F^{\rho\sigma}$ is the external electromagnetic force and $C[f]$ is the collision term which is given in the relaxation time approximation (RTA) as

$$C[f] = -\frac{p^\mu u_\mu}{\tau_i^B}\left(f_i^B - f_{\text{eq},i}^B\right), \qquad (2)$$

and in the BGK collision term as

$$C[f] = -\frac{p^\mu u_\mu}{\tau_i^B}\left(f_i^B - n_i^B n_{\text{eq},i}^{B\ -1} f_{\text{eq},i}^B\right), \qquad (3)$$

where τ_i^B is the relaxation time and $n_{\text{eq},i}^B$ is the equilibrium number density.

Charge and heat transport coefficient with BGK collision term

The electrical conductivity (σ_{el}^B) in the BGK collision term can be expressed as sum of the contribution due to the RTA type collision integral and a correction term as [1]

$$\sigma_{\text{el}}^B = \sigma_{\text{el}}^{B,\text{RT}} + \sigma_{el}^{B,\text{Corr}}, \qquad (4)$$

where

$$\sigma_{el}^{B,\text{RT}} = \frac{\beta}{\pi^2}\sum_i q_i^2 g_i |q_i B| \int dp_3 \frac{p_3^2\ \tau_i^B}{\omega_i^2} \times f_{\text{eq},i}^B (1 - f_{\text{eq},i}^B), \qquad (5)$$

$$\sigma_{el}^{B,\text{Corr}} = \frac{\beta}{\pi^2}\sum_i q_i^2 g_i^2 |q_i B| n_{\text{eq},i}^{B\ -1} \int dp_3 \frac{p_3}{\omega_i} \times f_{\text{eq,i}}^B \int_{p_3'} \frac{p_3'}{\omega_i}\ \tau_i^B(p_3')\ f_{\text{eq,i}}^B \times (1 - f_{\text{eq,i}}^B), \qquad (6)$$

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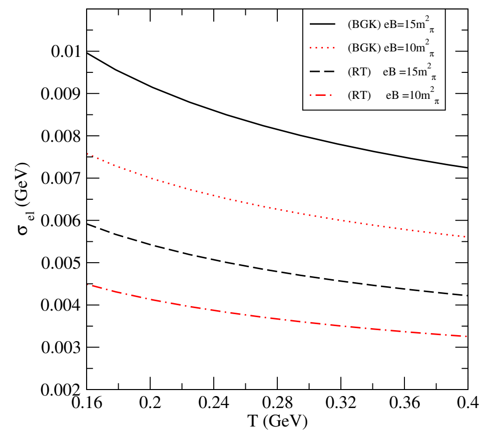

FIG. 1: Variation of electrical conductivity with temperature

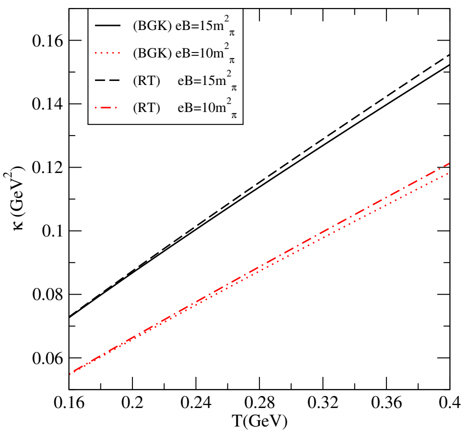

FIG. 2: Variation of thermal conductivity with temperature

Similarly the thermal conductivity (κ^B) can also be written as [1]

$$\kappa^B = \kappa^{\mathrm{B,RT}} + \kappa^{\mathrm{B,Corr}}, \tag{7}$$

where

$$\kappa^{\mathrm{B,RT}} = \frac{\beta^2}{2\pi^2}\sum_i g_i|q_iB|\int dp_3 \frac{p_3^2\ \tau_i^B}{\omega_i^2} \times(\omega_i - h_i^B)^2 f^B_{\mathrm{eq,i}}(1-f^B_{\mathrm{eq,i}}), \tag{8}$$

$$\kappa^{\mathrm{B,Corr}} = \frac{\beta^2}{2\pi^2}\sum_i g_i^2|q_iB|{n^B_{\mathrm{eq},i}}^{-1}\int dp_3\frac{p_3}{\omega_i} \times(\omega_i - h_i^B)f^B_{\mathrm{eq,i}}\int_{p_3'}\frac{p_3'\ \tau_i^B(p_3')}{\omega_i'} \times(\omega_i' - h_i^B)f^B_{\mathrm{eq,i}}(1-f^B_{\mathrm{eq,i}}). \tag{9}$$

In this study, we have computed the σ_{el}^B and κ^B as a function of temperature (in Fig 1 and 2) at $eB = 10m_\pi^2$ and $eB = 15m_\pi^2$. We have used the quasi-particle mass for i^{th} flavour as $m_i^2 = m_{i0}^2 + \sqrt{2}m_{i0}m_{iT,B} + m_{iT,B}^2$, which has been proposed in [2]. Here, m_{i0} is the current quark mass and $m_{iT,B}$ is the medium generated mass calculated as $m_{iT,B}^2 = \frac{g^2|q_iB|}{3\pi^2}\left[\frac{\pi T}{2m_i} - \ln(2)\right]$. We have noticed from figures. 1 and 2 that the σ_{el}^B decreases while κ^B increases with the temperature. The magnitude of the σ_{el}^B gets enhanced in the BGK collision term as compared to that of RTA type while behavior of κ^B gets reversed. It's magnitude is slightly greater in the RTA collision term. We have also observed that the magnitude of both the conductivities σ_{el}^B and κ^B increases as the strength of magnetic field rises. We have also studied the Wiedemann-Franz law which states that the ratio of the thermal conductivity to the electrical conductivity is directly proportional to the temperature *i.e* ($\kappa^B/\sigma_{el}^B = L_RT$) and the proportionality constant L_R is knows as *Lorentz number*. We have found that L_R increases with the temperature violating the Wiedemann-Franz law and its magnitude is reduced in the BGK collision term as compared to the RTA type.

References

[1] S. A. Khan and B. K. Patra, (2020), arXiv:2011.02682 [hep-ph]. (Accepted in Phys. Rev. D)

[2] V. M. Bannur, J. High Energy Phys. 09 (2007) 046.