Field theoretical conductivity of dense quark matter in presence of magnetic field

Sarthak Satapathy[1,2*]
[1]Indian Institute of Technology Bhilai, GEC Campus,
Sejbahar, Raipur - 492015, Chhattisgarh, India and
[2]National Institute of Science Education and Research, Jatni, Khurda - 752050

Snigdha Ghosh[3†]
Government General Degree College Kharagpur-II, Madpur,
Paschim Medinipur - 721149, West Bengal, India

Sabyasachi Ghosh[1‡]
[1]Indian Institute of Technology Bhilai, GEC Campus,
Sejbahar, Raipur - 492015, Chhattisgarh, India

In this work we have have studied the electrical conductivity σ of a dense quark matter that is anticipated to be present in the core of neutron stars and magnetars [1]. This highly dense matter is subjected to an intense background magnetic field of 10^{18} G [2]. To calculate σ we need to compute the spectral function of $U(1)$ vector currents $J^\mu(x)$ given by

$$\rho^{\mu\nu}(q) = \text{Im}\; i\int d^4x e^{iqx}\langle \mathcal{T}_c J^\mu(x)J^\nu(0)\rangle \quad (1)$$

where $J^\mu = \overline{\psi}\gamma^\mu\psi$ is the $U(1)$ vector current, ψ and $\overline{\psi}$ are the Dirac fields subjected to background magnetic field given by the four vector potential $\widetilde{A}^\mu = A^\mu + A^\mu_{ext}$, where A^μ_{ext} is the background magnetic field in z-direction and $\mathcal{T}_c$ is the time ordering operator. The spectral function in Eq.(1) is evaluated by employing the Dirac propagator in background magnetic field [3] which takes into account the *Landau quantization of energy*. In real time thermal field theory propagator get 2×2 matrix structure, whose 11 compent D^B_{11} can be used for calculating corresponding 11 component of one-loop self energy, from where spectral function can be obtained. Based on the Schwinger proper time formalism, one can get general form of propagator as [4]

$$D^B_{11}(k;m) = \sum_{l=0}^{\infty}(-1)^l e^{-2l} D_l \left\{\frac{-1}{k_\parallel^2 - m_l^2 + i\epsilon} - \xi_-(k_0)2\pi i\delta(k_\parallel^2 - m_l^2)\right\}\,, \quad (2)$$

where D_l carry rich anatomy with Laggure polynomials and gamma matrics [4]. $\xi_-(k_0) = \Theta(k_0)f_+ + \Theta(-k_0)f_-$ and $f_\pm = \{e^{(\omega_l\mp\mu)/T}+1\}^{-1}$ denotes the Fermi-Dirac thermal distribution functions for quark and anti-quark with energy $\omega_l = \{\vec{k}_\parallel^2 + m_l^2\}^{1/2}$ and $m_l = \{2leB + m^2\}^{1/2}$. Here perpendicular momentum is quantized as $\vec{k}_\perp^2 = 2leB$, where quantum number l (integer values) is known as Landau level. The momentum parallel to magnetic field $\vec{k}_\parallel$ remain un-quantized variable and we define $k_\parallel^2 = k_0^2 - \vec{k}_\parallel^2$.

By employing Kubo formulas for conductivity in an external magnetic field we get the electrical conductivity

$$\sigma^\upsilon_B = \mathcal{P}^\upsilon_{\mu\nu}\sigma^{\mu\nu} \;\; ; \;\; \upsilon \in \{\parallel, \perp, \times\} \quad (3)$$

where the projectors $\mathcal{P}^\upsilon_{\mu\nu}$ contract with $\sigma^{\mu\nu}$ to give σ^υ and $\sigma^{\mu\nu}$ is given by

$$\sigma^{\mu\nu} = \lim_{q_0,\vec{q}\to\vec{0}} \frac{\rho^{\mu\nu}(q_0,\vec{q})}{q_0}. \quad (4)$$

*Electronic address: sarthaks@iitbhilai.ac.in
†Electronic address: snigdha.physics@gmail.com
‡Electronic address: sabyaphy@gmail.com

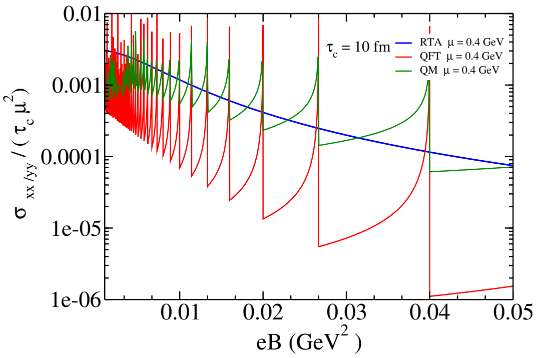

FIG. 1: $\sigma^{\perp}/(\tau_c\mu^2)$ vs eB

Cold and dense QCD matter exists in the region where $T << \mu$ which leads to Fermi-Dirac distriution function of particles/antiparticles $f_{\pm}(x)$ behave like the Heaviside theta function $\Theta(x)$ given by $\lim_{\beta\to\infty}\frac{1}{e^{\beta(\omega_l\pm\mu)}+1} = \Theta(\mu\pm\omega_l)$. Evaluating Eq.(4) by using the rich structure of propagator (2), one can get $\sigma^{\parallel}$ and $\sigma^{\perp}$ components of conductivity. In Fig.(1), we have shown the variation of the dimensionless quantity $\sigma^{\perp}/(\tau_c\mu^2)$ (red solid line) with magnetic field and we call it quantum field theoretical (QFT) curve, which is compared with the results (blue and green solid lines) of Ref. [5] which is based on Relaxation Time approximation (RTA). Imposing quantum mechanical (QM) changes due to Landau quantization [5] in RTA expression(blue solid line), we will get QM curve (green solid line).

We see that the QM and QFT conductivity oscillates with magnetic field which is reminescent of a similar effect observed in condensed matter physics known as *Shubnikov de-Haas effect* or SdH oscillations [6] in which the electrical conductivity of a material oscillates with magnetic field [6]. It is an effect which has deep connection with Landau quantization of energies and is observed at low temperatures and intense magnetic fields. In present work, we notice a difference between QFT and QM curves, which probably indicates that a rich field theoretical information is hidden in QFT or Kubo expressions. Finding a clear interpretations of this field theoretical conduction of dense matter is still under progress and soon it will be comunicated in journal article.

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