Relativistic resistive dissipative magnetohydrodynamics from the relaxation time approximation

Ankit Kumar Panda[1],* Ashutosh Dash[1], Rajesh Biswas [1], and Victor Roy[1]
[1]*National Institute of Science Education and Research, Bhubaneswar, HBNI, Jatni, 752050, India*

Introduction

The discovery of the Quark-Gluon-Plasma (QGP)produced in high-energy heavy-ion collisions is a near-perfect fluid is primarily based on phenomenological studies using relativistic viscous hydrodynamics. A strong transient electromagnetic field is also produced in the initial stage of high-energy heavy-ion collisions mainly by the spectators whose effects are of great importance and need a detailed study. The finite electrical conductivity of the QGP and the ambient intense electromagnetic fields strongly suggest that the most appropriate framework for this case is relativistic resistive viscous magnetohydrodynamics. Here we have derived a second order evolution equation of the viscous stresses in presence of electromagnetic field along with their transport co-efficients from the kinetic theory using relaxation time approximation for the collision kernel. Also we have calculated various components of conductivity in this case.

Relativistic Magnetohydrodynamics Equations

Due to space limitation here we discuss the essential equations briefly. The magnetohydrodynamics equations consists of energy-momentum conservation equations for fluid and electromagnetic field and the Maxwell's equations. In presence of the electromagnetic field, there exists an external force on charged fluid and the conservation equation for energy-momentum tensor of the fluid takes the following form:

$$\partial_\mu T_f^{\mu\nu} = F^{\nu\lambda} J_\lambda \qquad (1)$$

*Electronic address: `ankitkumar.panda@niser.ac.in`

where $F^{\mu\nu} = E^\mu u^\nu - E^\nu u^\mu + \epsilon^{\mu\nu\alpha\beta} u_\alpha B_\beta$. E^μ B_β are four component of electric and magnetic field respectively with J^μ being the fluid charge density. $F^{\mu\nu}$ obeys the maxwell's equation:

$$\partial_\mu F^{\mu\nu} \;=\; J^\nu. \qquad (2)$$

Formalism and Boltzmann equation

A. Boltzmann Equation

The relativistic Boltzmann equation(RBE) is given by

$$p^\mu \partial_\mu f + qF^{\mu\nu} p_\nu \frac{\partial}{\partial p^\mu} f = C[f]. \qquad (3)$$

where f is the distribution function, q is the electric charge and $C[f]$ is the collision kernel. Here we take the collision kernel as $C[f] = -\frac{u \cdot p}{\tau_c}\delta f$ where τ_c is the relaxation time and $\delta f = f - f_0$ is the deviation from the local-equilibrium distribution function f_0.

B. First and second order correction in distribution function

Here we use the techniques similar to Ref. [1] in order to calculate δf corrections. Eq. (3) can be written as a power series expansion of the following form

$$f = \sum_{n=0}^{\infty} (-1)^n \left(\frac{\tau_c}{u.p}\right)^n \left(p^\mu \partial_\mu + qF^{\mu\nu} p_\nu \frac{\partial}{\partial p^\mu}\right)^n f_0. \qquad (4)$$

Here we expand the above to get the distribution function in terms of the knudsen number $Kn = \tau_c \partial_\mu$,$\chi = qB\tau_c/T$ and $\xi = qE\tau_c/T$ as the expansion parameter. Now truncating it upto second order we get

$$f = f_0 + \delta f^{(1)} + \delta f^{(2)}, \qquad (5)$$

where

$$\delta f^{(1)} = -\frac{\tau_c}{u\cdot p}\left(p^\mu\partial_\mu f_0 + \beta q E^\nu p_\nu f_0\tilde{f}_0\right),$$

$$\delta f^{(2)} = -\frac{\tau_c}{u\cdot p}\left(p^\mu\partial_\mu + qF^{\mu\nu}p_\nu\frac{\partial}{\partial p^\mu}\right)(\delta f)^1 .$$

Similarly the correction for anti-particles ($\delta\bar{f}$) are calculated.

C. First order evolution equations

Following are the first order constitutive relations taking the $(\delta f)^1$:

$$\Pi_{(1)} = -\tau_c\beta_\Pi\theta, \tag{6}$$

$$V^\mu_{(1)} = \tau_c\beta_V\left(\nabla^\mu\alpha + \beta q E^\mu\right), \tag{7}$$

$$\pi^{\mu\nu}_{(1)} = 2\tau_c\beta_\pi\sigma^{\mu\nu}. \tag{8}$$

D. Second order evolution equations

Here we evaluate the second order equations for dissipative stresses.

$$\begin{aligned}\frac{\Pi}{\tau_c} &= -\dot{\Pi} - \delta_{\Pi\Pi}\Pi\theta + \lambda_{\Pi\pi}\pi^{\mu\nu}\sigma_{\mu\nu} - \tau_{\Pi V}V\cdot\dot{u}\\ &- \lambda_{\Pi V}V\cdot\nabla\alpha - l_{\Pi V}\partial\cdot V - \beta_\Pi\theta - qB\lambda_{\Pi VB}\\ &\quad b^{\mu\beta}V_\beta V_\mu - q^2\tau_c\chi_{\Pi EE}E^\mu E_\mu + \tau_c\tau_{\Pi VB}\\ &\quad \dot{u}_\alpha qBb^{\alpha\beta}V_\beta - q\delta_{\Pi VB}\nabla_\mu\left(\tau_c Bb^{\mu\beta}V_\beta\right)\end{aligned}$$

$$\begin{aligned}\frac{\pi^{\mu\nu}}{\tau_c} &= -\dot{\pi}^{\langle\mu\nu\rangle} + 2\beta_\pi\sigma^{\mu\nu} + 2\pi_\gamma^{\langle\mu}\omega^{\nu\rangle\gamma} - \tau_{\pi\pi}\\ &\quad \pi_\gamma^{\langle\mu}\sigma^{\nu\rangle\gamma} - \delta_{\pi\pi}\pi^{\mu\nu}\theta + \lambda_{\pi\Pi}\Pi\sigma^{\mu\nu} - \tau_{\pi V}\\ &\quad V^{\langle\mu}\dot{u}^{\nu\rangle} - \tau_c qB\tau_{\pi VB}\dot{u}^{\langle\mu}b^{\nu\rangle\sigma}V_\sigma + \lambda_{\pi V}\\ &\quad V^{\langle\mu}\nabla^{\nu\rangle}\alpha - l_{\pi V}\nabla^{\langle\mu}V^{\nu\rangle} + \delta_{\pi B}\Delta^{\mu\nu}_{\eta\beta}qB\\ &\quad b^{\gamma\eta}g^{\beta\rho}\pi_{\gamma\rho} - qB\lambda_{\pi VB}V_\gamma b^{\gamma\langle\mu}V^{\nu\rangle} - q\delta_{\pi VB}\\ &\quad \nabla^{\langle\mu}\left(\tau_c B^{\nu\rangle\gamma}V_\gamma\right) + q^2\tau_c\chi_{\pi EE}\Delta^{\mu\nu}_{\sigma\rho}E^\sigma E^\rho .\end{aligned}$$

$$\begin{aligned}\frac{V^\mu}{\tau_c} &= -\dot{V}^{\langle\mu\rangle} - V_\nu\omega^{\nu\mu} + \lambda_{VV}V^\nu\sigma^\mu_\nu - \delta_{VV}V^\mu\theta\\ &\quad +\lambda_{V\Pi}\Pi\nabla^\mu\alpha - \lambda_{V\pi}\pi^{\mu\nu}\nabla_\nu\alpha - \tau_{V\pi}\pi^\mu_\nu\dot{u}^\nu\\ &\quad -qB\delta_{VB}b^{\mu\gamma}V_\gamma + \tau_{V\Pi}\Pi\dot{u}^\mu + \beta_V\nabla^\mu\alpha\\ &\quad +l_{V\pi}\Delta^{\mu\nu}\partial_\gamma\pi^\gamma_\nu - l_{V\Pi}\nabla^\mu\Pi + \tau_c qBl_{V\pi B}\\ &\quad b^{\sigma\mu}\partial^\kappa\pi_{\kappa\sigma} - q\tau_c\lambda_{VVB}Bb^{\gamma\nu}V_\nu\sigma^\mu_\gamma + \tau_c qB\\ &\quad \tau_{V\Pi B}b^{\gamma\mu}\Pi\dot{u}_\gamma - \tau_c qBl_{V\Pi B}b^{\gamma\mu}\nabla_\gamma\Pi - q\tau_c\\ &\quad \delta_{VVB}Bb^{\mu\nu}V_\nu\theta - q\tau_c\rho_{VVB}Bb^{\gamma\nu}V_\nu\omega^\mu_\gamma\\ &\quad +\chi_{VE}qE^\mu + q\Delta^\mu_\alpha\chi_{VE}D\left(\tau_c E^\alpha\right) - q\tau_c\rho_{VE}\\ &\quad E^\mu\theta - q\tau_{VVB}\Delta^\mu_\gamma D\left(\tau_c Bb^{\gamma\nu}V_\nu\right).\end{aligned}$$

We have found out the Navier-Stokes limit of the above second order evolution equations and also different components of conductivity that are present.

$$\begin{aligned}\sigma_E^{\parallel} &= q^2\tau_c\beta\beta_V,\\ \sigma_E^{\perp} &= \frac{q^2\tau_c\beta\beta_V}{1+\left(qB\tau_c\delta_{VB}\right)^2},\\ \sigma_E^{\times} &= \frac{q^3B\tau_c^2\beta\beta_V\delta_{VB}}{1+\left(qB\tau_c\delta_{VB}\right)^2}.\end{aligned} \tag{9}$$

Conclusion

In this work, we derive the second-order relativistic resistive dissipative magnetohydrodynamics equations using the relaxation time approximation of the collision kernel in the relativistic Boltzmann equation along with the novel transport coefficients originating due to the coupling of electromagnetic field and usual dissipative forces. Different components of electric conductivity have also been derived which obeys the kinetic version of the Wiedemann-Franz law. We wish to further extend the current formulation to curved spacetime, which is relevant for cosmological problems.

Acknowledgments

AP acknowledges the CSIR-HRDG financial support. AD and VR acknowledge support from the DAE, Govt. of India. RB and VR acknowledge financial support from the DST Inspire faculty research grant (IFA-16-PH-167), India.

References

[1] A. K. Panda et.al Relativistic non-resistive viscous magnetohydrodynamics from the kinetic theory: A relaxation time approach, J. High Energy Phys. 03 (2021) 216.