Local gauge invariance and dynamics of a non-equilibrium system close to critical region of phase transition

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In this work we calculate the time evolution of local gauge invariant field theoretical model, comprising of a scalar field coupled to vector gauge field. Assuming a linear relationship between phase angles $\alpha(x)$ at two closely separated space-time points x and $x' = x - \delta$, with $0 < \delta < 1$, we obtain an explicit dependence of scalar field $\phi(x)$ at x and x' in terms of Wilson-line variable. Using the modified value of field $\phi(x')$ we evaluate the effective coupling of this system in dimension $d < 4$ near the critical region. In the mean-field approximation, we found that the scalar self coupling λ at Wilson-Fisher fixed point of this system is modified as $\lambda* = \lambda_{WF}/t^4$, where $\lambda_{WF} = \frac{16\pi^2}{3}(d-4)$ and t is the time of evolution. With this modified coupling we found that the density of active states for this system behave as $\Omega \propto \frac{1}{t^4}$.

Introduction: The study of non-equilibrium system and the associated phase transition has been at the forefront of the research and recently have gained wide attention due to applicability in heavy ion collisions. Systems that are slightly deviated from equilibrium are well studied. However for systems that are in a state with large deviation from equilibrium have not been very well understood in terms of first principles. In this article we show that the time evolution of a system can be understood using local gauge invariance. We consider a field theoretical model of a scalar field coupled to vector gauge field having well defined local gauge invariance. Invoking local gauge invariance breaking in an infinitesimal region between two space-time points x and $x' = x - \delta$, we show that the effective coupling of this system in the vicinity of critical point in dimension $d < 4$ satisfies $\lambda* = \lambda_{WF}/t^4$, where λ_{WF} is the coupling at Wilson-Fisher fixed point given by $\lambda_{WF} = \frac{16\pi^2}{3}(4-d)$. Further we show that the density $\Omega(E)$ of active states satisfy $\Omega(E) \propto \frac{1}{t^4}$ in the critical region.

Model: Consider a Lagrangian

$$\mathcal{L} = \frac{1}{2}\left(D_\mu\phi\right)^2 + \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\phi^4 + \frac{1}{4}F_{\mu\nu}F^{\mu\nu}, \qquad (1)$$

with D_μ a covariant derivative involving a coupling with vector gauge field A_μ. This Lagrangian has well defined gauge invariance that involves shifting of field $\phi(x)$ and vector gauge field $A_\mu(x)$ simultaneously by phase rotation angle $\alpha(x)$.

Consider two infinitesimally separated space-time points x and $x' = x - \delta$, with $\delta > 0$. Assuming a linear relationship between phase angles at these space-time points defined as

$$\alpha(x') = (I + R)\,\alpha(x), \tag{2}$$

where R is some rotation matrix, the Lagrangian still is gauge invariant but with the condition on the gauge field

$$A_\mu(x') = A_\mu(x') - \frac{I+R}{eb^n}\partial_\mu \alpha(x), \tag{3}$$

where $b = 1 - \frac{\delta}{x}$. Now for $n > 1$ and $b < 1$ a situation can arise when first and second term on right hand side can become equal. This condition allows one to express rotation angle $\alpha(x)$ as

$$\alpha(x) = \frac{eb^n}{I+R}\left[\int A_\mu(x)dx^\mu - n\int \delta \cdot \partial A_\mu(x)dx^\mu - \cdots\right]. \tag{4}$$

Now imposing condition (2) on the field $\phi(x')$ we have

$$\phi(x') = \left[1 - \delta\cdot\partial - \iota K eb^n \int A_\mu(x)\cdot dx^\mu - \cdots\right]\phi(x), \tag{5}$$

with $K = \frac{R+R^2}{I+R}$.

Now we make an attempt to calculate the behaviour of this system at critical point. For this we integrate high momentum modes and follow same method as L. Kadanoff, K. G. Wilson and others [].

Consider the path integral in d dimensions

$$Z = \int [D\phi]\; e^{-\int \mathcal{L}\, d^dx}, \tag{6}$$

where $\mathcal{L}$ is defined in (1). Now using $\phi \to \phi + \hat{\phi}$, with $\hat{\phi}$ corresponding to high momentum mode with a non-zero value in the momentum range $b'\Lambda \le k \le \Lambda$. With this transformation one can write (6) in the form

$$Z = \int [D\phi]\, e^{-\int \mathcal{L}(\phi)} \int [D\hat{\phi}] e^{-\int d^dx\left[\frac{1}{2}(D_\mu\hat{\phi})^2 + \frac{1}{2}m^2\hat{\phi}^2 + \lambda\left(\frac{1}{6}\phi^3\hat{\phi} + \frac{1}{4}\phi^2\hat{\phi}^2 + \frac{1}{6}\phi\hat{\phi}^3 + \frac{1}{4!}\hat{\phi}^4\right)\right]}. \tag{7}$$

Now we attempt to write (7) in the form

$$Z = \int [D\phi]_{b'\Lambda}\, exp\left(-\int d^dx \mathcal{L}_{eff}\right). \tag{8}$$

Taking all other terms except kinetic term as perturbation we integrate on the field components $\hat{\phi}$.