Impact of $\overline{\psi}\psi\phi$ type interactions on the electrical conductivity of Fermi fluids in strong magnetic fields

Sarthak Satapathy[1,2*]
[1] Indian Institute of Technology Bhilai, GEC Campus, Sejbahar, Raipur - 492015, Chhattisgarh, India and
[2] School of Physical Sciences, National Institute of Science Education and Research, HBNI, Jatni, Khurda - 752050

Intensely magnetized objects such as Neutron stars (NS) and magnetars have a particle profile of which hadrons form the outermost layer and the anticipated quark matter (QM) [1] in the core. The strength of the magnetic field of this order has an effect on thermodynamic and transport properties [2, 3] of the Fermi fluid by interfering with the thermal vacuum thus creating an anisotropy in the medium with $k^\mu_\parallel = (k_0, 0, 0, -k_z)$, $k^\mu_\perp = (0, -k_x, -k_y, 0)$ when the magnetic field points in z-direction following which Landau quantized energies are given by $\omega_l = \sqrt{k_z^2 + m^2 + 2leB}$, l being the Landau level, B being the external magnetic field. In this work we have studied the effect of Yukawa type interactions given by $\overline{\psi}\psi\phi$ where ϕ is the scalar field, on electrical conductivities σ^υ, $\upsilon \in \{\parallel, \perp\}$ [3] of cold and dense matter in strong magnetic field that falls in the regime $\mu >> T$ of QCD phase diagram, where μ is the baryon chemical potential and T is the temperature. Electrical conductivities in the $\mu >> T$ region with background magnetic field [3, 4] in z-direction is given by

$$\sigma^{||} = \left\{\frac{e^2 eB\sqrt{\mu^2 - m^2}}{2\pi^2\Gamma\mu} + \sum_{l=1}^{l_{max}} \frac{e^2 eB\sqrt{\mu^2 - m^2 - 2leB}}{\pi^2\Gamma\mu}\right\}\Theta(\mu - m_l) \quad (1)$$

$$\sigma^\perp = e^2\frac{(eB)^2}{2\pi^2}\frac{\Gamma}{\left\{\Gamma^2 + (\mu - \sqrt{\mu^2 - 2eB})^2\right\}\left\{\sqrt{\mu^2 - 2eB}\right\}} \sum_{l=1}^{l_{max}} \Theta(\mu - m_l)\frac{2l-1}{\sqrt{\mu^2 - m_l^2}} \quad (2)$$

where $m_l = \sqrt{k_z^2 + m^2 + 2leB}$, Γ is the thermal decay width, $l_{max} = \frac{\mu^2 - m^2}{2eB}$ is the maximum of allowed Landau levels and $\Theta(\mu - m_l)$ is the Heaviside step function. For the strong magnetic field case the contribution of lowest Landau level yields the following result

$$\widetilde{\sigma}^\parallel = \frac{e^2(eB)^2\Gamma\ \Theta(\mu - m_1)}{2\pi^2\left\{\Gamma^2 + (\mu - \sqrt{\mu^2 - 2eB})^2\right\}\left\{\mu^2 - 2eB\right\}\left\{\sqrt{\mu^2 - m_1^2}\right\}}$$
$$\widetilde{\sigma}^\perp = \frac{e^2 eB\sqrt{\mu^2 - m^2}}{2\pi^2\Gamma\mu}\Theta(\mu - m) + \frac{e^2 eB\sqrt{\mu^2 - m_1^2}}{\pi^2\Gamma\mu}\Theta(\mu - m_1) \quad (3)$$

where m_1 is m_l at $l = 1$. In Eq.(3) Γ can be calculated from interaction lagrangian. To proceed with the calculation we employ the thermomagnetic Dirac propagator [5]

$$iS(k) = ie^{\frac{-k_\perp^2}{eB}}\sum_{l=0}^{\infty}\frac{(-1)^l D_l(e,B,k)}{k_\parallel^2 - m^2 - 2leB}, \quad \text{where}$$
$$D_l(e,B,k) = (\not{k}_\parallel + m)\Big\{(1 - i\gamma^1\gamma^2)L_l(2k_\perp^2/eB) - (1 + i\gamma^1\gamma^2)L_{l-1}(2k_\perp^2/eB)\Big\} - 4\not{k}_\perp L_{l-1}(2k_\perp^2/eB), \quad (4)$$

$L_n(x)$ being the generalized Laguerre polyno-

*Electronic address: sarthaks@iitbhilai.ac.in

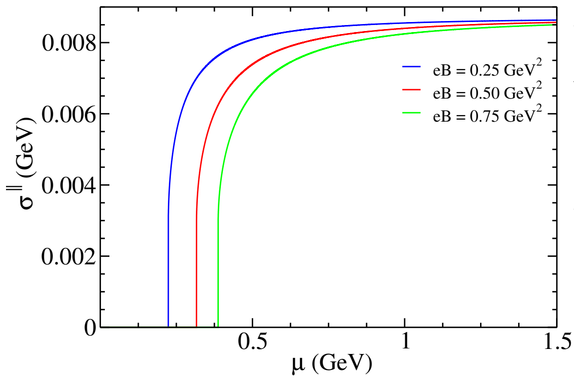

FIG. 1: $\sigma^{\parallel}$ vs μ at fixed values of magnetic field 0.025 , 0.050 & 0.075 GeV^2

mial represented by

$$(1-z)^{-(l+1)} e^{\frac{xz}{z-1}} = \sum_{l=0}^{\infty} L_l^n(x) z^n \qquad (5)$$

and $\not{q} = \gamma^\mu q_\mu$, γ^μ are the gamma matrices. In strong magnetic field Eq.(4) reads

$$iS(k) = ie^{\frac{-k_\perp^2}{eB}} \frac{\not{k}_\parallel + m}{k_\parallel^2 - m^2} \{1 - i\gamma^1\gamma^2\}. \qquad (6)$$

Employing Eq.(6), Γ [6] is given by

$$\Gamma = \frac{g^2 eB}{2\pi M}\left[1 - \frac{4m^2}{M^2}\right]^{1/2} \left[1 - f(M/2)\right]\Theta(M^2 - 4m^2) \qquad (7)$$

where M is the mass of the decaying scalar, f is the Fermi-Dirac distribution function. We observe that the decay width depends on magnetic field. Thus using Eq.(7) we obtain $\sigma^{\parallel,\perp}$, where $\sigma^{\parallel}$ is plotted with μ in Fig.(1) .It is observed that with increase in μ the conductivity $\sigma^{\parallel}$ attains non-zero values after the threshold μ of $2eB$ is attained. At large values of μ the electrical conductivity saturates and becomes independent of μ and B.

References

[1] E. Annala, T. Gorda, A. Kurkela, J. Nättilä and A Vuorinen, *Evidence for quark-matter cores in massive neutron stars*, Nature Physics, **16**, 2020, 907-910.

[2] S. Ghosh , S. Ghosh, *One-loop Kubo estimations of the shear and bulk viscous coefficients for hot and magnetized Bosonic and Fermionic systems*, Phys. Rev. D **103**, 096015, (2020).

[3] S. Satapathy, S. Ghosh, S. Ghosh, *Kubo formalism of electrical conductivity of relativistic fluid in presence of magnetic field*, arXiv:2104.03917 [hep-ph].

[4] J. Dey, A. Bandyopadhyay, A. Gupta, N. Pujari, S. Ghosh, *Electrical conductivity of strongly magnetized dense quark matter – possibility of quantum hall effect*, arXiv:2103.15364 [hep-ph].

[5] T-K Chyi, C-W Hwang, W. F. Kao, G-L Lin, Kin-Wang Ng, Jie-Jun Tseng, *Weak-field expansion for processes in a homogeneous background magnetic field*, Phys. Rev. D **62**, 105014, (2000).

[6] A. Bandyopadhyay, R.L.S. Farias, R.O. Ramos, *Effect of the magnetized medium on the decay of neutral scalar bosons*, Phys. Rev. D **98**, 076007, (2018).