Effective relaxation times and decay width of cold and dense Fermi fluids in a magnetized background

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Off central heavy ion collisions are known to generate magnetic fields that change the isotropic properties of the transport coefficients [1] of the fireball medium. Cold and dense neutron stars (NS) have been long anticipated to contain a magnetized quark matter (QM) core [2]. Electrical conductivity σ [3] is one of the transport coefficients whose anisotropic property and it's implications have been studied in this work. Electrical conductivities σ^{υ} , $\upsilon \in \{\parallel, \perp\}$ in relaxation time approximation (RTA) [4] is given by

$$\sigma^{\parallel} = \frac{e^2}{2\pi^2}\sum_{l=0}^{l_{max}} \alpha_l eB\tau_c \frac{\sqrt{\mu^2-2leB-m^2}}{\mu}$$
$$\sigma^{\perp} = \frac{e^2}{\pi^2}\sum_{l=1}^{l_{max}} \frac{(eB)^2 l}{\mu\sqrt{\mu^2-2leB-m^2}}\frac{\tau_c}{1+\frac{\tau_c^2}{\tau_B^2}} \quad (1)$$

where the Fermi-Dirac distribution in the $\mu >> T$ limit has been approximated as Heaviside theta function $\Theta(\mu-\omega_l)$ where μ is the baryon chemical potential , τ_c is the relaxation time, $\tau_B = \frac{\omega_l}{eB}$, $\omega_l = \sqrt{k_z^2+m^2+2leB}$ is the Landau quantized energy, l is the Landau level, B is the external magnetic field and $l_{max} \leq \frac{\mu^2-m^2}{2eB}$. Alternatively one can use Kubo formulas [5] for σ^{υ} which can be calculated by taking the derivative of the static limit of the spectral function of currents $J^{\mu}(x)$, given by $\rho^{\mu\nu}(q)$ which for a $U(1)$ gauge theory is given by

$$\rho^{\mu\nu}(q) = \text{Im}\, i\int d^4x e^{iqx}\left\langle J^{\mu}(x)J^{\nu}(0)\right\rangle_R \quad (2)$$

that is evaluated on the Schwinger-Keldysh contour [1, 3]. Landau quantization is introduced at the level of the propagator [6] which keeps the magnetic field dependent transport profile, a fundamental property of the fluid. By computing σ^{υ} from Kubo formulas [3] and employing the $\mu >> T$ approximation of distribution function we arrive at

$$\sigma^{||} = \left\{\frac{e^2eB\sqrt{\mu^2-m^2}}{2\pi^2\Gamma\mu} + \sum_{l=1}^{l_{max}}\frac{e^2eB\sqrt{\mu^2-m^2-2leB}}{\pi^2\Gamma\mu}\right\}\Theta(\mu-m_l) \quad (3)$$

$$\sigma^{\perp} = e^2\frac{(eB)^2}{2\pi^2}\frac{\Gamma}{\left\{\Gamma^2+(\mu-\sqrt{\mu^2-2eB})^2\right\}\left\{\sqrt{\mu^2-2eB}\right\}} \sum_{l=1}^{l_{max}}\Theta(\mu-m_l)\frac{2l-1}{\sqrt{\mu^2-m_l^2}} \quad (4)$$

where Γ is the thermal decay width related with τ_c by $\Gamma = \tau_c^{-1}$ and $m_l = \sqrt{m^2+2leB}$. The origin of the anisotropic behaviour is connected with the background magnetic field. This in turn changes the relaxation times which we call *effective relaxation time* of the fluid. From Eq.(1) and Eqs.(3-4) we extract the effective relaxation times as follows

$$\tau_c^{\parallel} = \tau_c \quad , \quad \Gamma = (\tau_c^{\parallel})^{-1}$$
$$\tau_c^{\perp} = \tau_c/(1+\frac{\tau_c^2}{\tau_B^2}) \quad ,$$
$$(\Gamma^{\perp})^{-1} = \Gamma/(\Gamma^2+(\mu-\sqrt{\mu^2\pm 2eB})^2) \quad (5)$$

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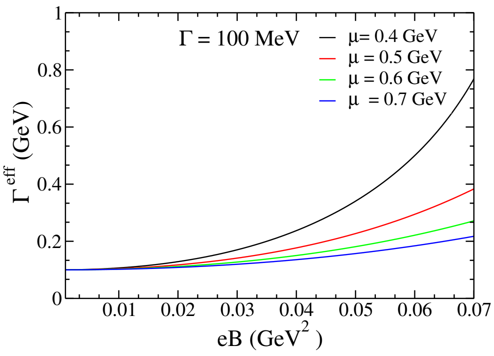

FIG. 1: Γ^{eff} vs eB at $\mu = 0.4, 0.5$ GeV

where τ_c is the relaxation time in the absence of background magnetic field, $\tau_c^{\parallel}$ and $\tau_c^{\perp}$ are the effective relaxation times in the presence of background magnetic field, $\Gamma^{\perp}$ is the *effective decay width* magnetized medium, Γ is the decay width in thermal vacuum. From Eq.(5) we get

$$\Gamma^{\perp} = \Gamma^{eff} = \frac{\Gamma^2 + (\mu - \sqrt{\mu^2 \pm 2eB})^2}{\Gamma} \quad (6)$$

where we have taken $\Gamma^{\perp}$ to be the effective decay width Γ^{eff}, that tells us about the change in decay width with μ and eB. To illustrate the effect of magnetic field that it has on nucleons in NS, magnetars and other compact stars we have plotted in Fig.(1) $\Gamma^{\perp}$ vs eB at $\Gamma = 0.1$ GeV and $\mu = 0.5$ GeV where we see that with increase in strength of magnetic field the decay width of fermions increases. We also note that with increase in μ, Γ^{eff} shows a decrease compared to lower values of μ. For $\mu^2 >> 2eB$, Γ^{eff} can be approximated as

$$\Gamma^{eff} \approx \Gamma + \frac{\mu^2}{\Gamma}\left\{\frac{eB}{\mu^2} - \frac{e^2B^2}{2\mu^4} + ...\right\} \quad (7)$$

which shows that $\Gamma^{\perp}$ undergoes a lesser deviation at high μ. A systematic study of decay processes occuring in compact stars taking into account Yukawa type interaction will be discussed in the future.

References

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