Neutron and Proton Pairing Energies of Even-Even Nuclei

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Introduction

The neutron and proton pairing energies in atomic nuclei are the manifestation of the fact that the ground state masses of odd-A and odd-odd nuclides is systematically higher as compared to neighboring even-even nuclides [1]. Many theoretical efforts have been made to understand and calculate the neutron and proton pairing energies using different theoretical approaches [2-5]. Additionally, several finite difference mass formulae used to calculate neutron and proton pairing energies in even-even, odd-A and odd-odd nuclei have also been suggested [1,5,6]. In present paper, we calculated, the neutron and proton pairing energies using measured ground state masses [7] in finite difference equations derived using Taylor series expansion of nuclear masses up to forth order derivative. We also suggested a functional form that could be used to estimate the average neutron and proton pairing energies.

Methodology

In the present work, the neutron and proton pairing energies are calculated using finite difference mass formulae derived from Taylor series expansions of nuclear masses [1]. The viability of these formulae is based on the assumption that, the nuclear ground state mass changes smoothly as a function of Z and N and hence the regions having departure from smoothness of masses, are excluded in the present calculations.

The smooth mass surface in the neighborhood of the mass of interest can be represented by Taylor's series expansion of $m(Z,N)$ in power of nucleon number as [1]:

$$M(Z,N) = m(Z,N_o) + (N-N_o)\frac{\partial m(Z,N_o)}{\partial N} + \frac{1}{2!}(N-N_o)^2\frac{\partial^2 m(Z,N_o)}{\partial N^2} + \frac{1}{3!}(N-N_o)^3\frac{\partial^3 m(Z,N_o)}{\partial N^3} + \frac{1}{4!}(N-N_o)^4\frac{\partial^4 m(Z,N_o)}{\partial N^4} + ...D(Z,N) \quad (1)$$

The correction term $D(Z,N)$ to the nuclear mass surface due to proton and neutron pairing energies is given by:

$$D(Z,N) = \Delta_n + \Delta_p - \delta \qquad (odd-odd)$$
$$D(Z,N) = \Delta_n \qquad (odd-N)$$
$$D(Z,N) = \Delta_p \qquad (odd-Z)$$
$$D(Z,N) = 0 \qquad (even-even)$$

To obtain seven-point finite difference mass formula for neutron pairing energy (Δ_n), we solved following set of simultaneous equations:

$$M(Z,N+2) = m(Z,N) + 2\frac{\partial m}{\partial N} + 2\frac{\partial^2 m}{\partial N^2} + \frac{4}{3}\frac{\partial^3 m}{\partial N^3} + \frac{2}{3}\frac{\partial^4 m}{\partial N^4}$$
$$M(Z,N+1) = m(Z,N) + \frac{\partial m}{\partial N} + \frac{1}{2}\frac{\partial^2 m}{\partial N^2} + \frac{1}{6}\frac{\partial^3 m}{\partial N^3} + \frac{1}{24}\frac{\partial^4 m}{\partial N^4} + \Delta_n$$
$$M(Z,N) = m(Z,N)$$
$$M(Z,N-1) = m(Z,N) - \frac{\partial m}{\partial N} + \frac{1}{2}\frac{\partial^2 m}{\partial N^2} - \frac{1}{6}\frac{\partial^3 m}{\partial N^3} + \frac{1}{24}\frac{\partial^4 m}{\partial N^4} + \Delta_n$$
$$M(Z,N-2) = m(Z,N) - 2\frac{\partial m}{\partial N} + 2\frac{\partial^2 m}{\partial N^2} - \frac{4}{3}\frac{\partial^3 m}{\partial N^3} + \frac{2}{3}\frac{\partial^4 m}{\partial N^4}$$
$$M(Z,N+3) = m(Z,N) + 3\frac{\partial m}{\partial N} + \frac{9}{2}\frac{\partial^2 m}{\partial N^2} + \frac{9}{2}\frac{\partial^3 m}{\partial N^3} + \frac{27}{8}\frac{\partial^4 m}{\partial N^4} + \Delta_n$$
$$M(Z,N-3) = m(Z,N) - 3\frac{\partial m}{\partial N} + \frac{9}{2}\frac{\partial^2 m}{\partial N^2} - \frac{9}{2}\frac{\partial^3 m}{\partial N^3} + \frac{27}{8}\frac{\partial^4 m}{\partial N^4} + \Delta_n$$

After solving above set of equations, we obtained seven-point finite difference mass formulas for neutron pairing energies (Δ_n) and for proton pairing energies (Δ_p) respectively as:

$$\Delta_n = \frac{3}{16}\left\{\begin{matrix}\frac{5}{2}\left[M(Z,N+1)+M(Z,N-1)\right]- \\ \left[M(Z,N+2)+M(Z,N-2)\right]+ \\ \frac{1}{6}\left[M(Z,N+3)+M(Z,N-3)\right]-\frac{10}{3}M(Z,N)\end{matrix}\right\}$$

$$\Delta_p = \frac{3}{16}\left\{\begin{matrix}\frac{5}{2}\left[M(Z+1,N)+M(Z-1,N)\right]- \\ \left[M(Z+2,N)-M(Z-2,N)\right]+ \\ \frac{1}{6}\left[M(Z+3,N)+M(Z-3,N)\right]-\frac{10}{3}M(Z,N)\end{matrix}\right\}$$

Results and Discussion

The present version of seven-point finite difference mass formulae is used to calculate the neutron pairing (Δ_n) and proton pairing (Δ_p) energies of 315 and 257 even-even nuclides, respectively. The experimental masses used in present calculations are taken from recently updated AME-2020 [7]. The variation of Δ_n and Δ_p as a function of mass number (A) are shown in Fig. 1 and Fig 2, respectively.

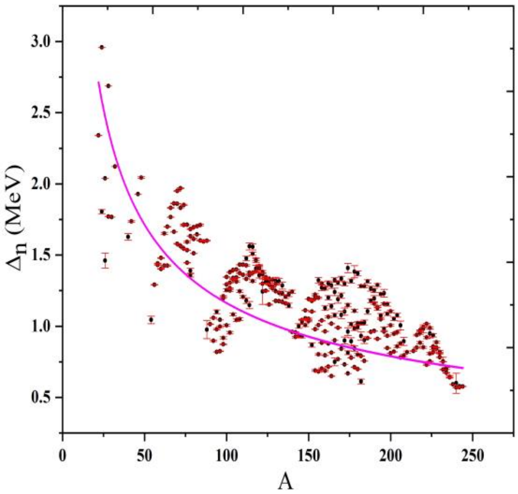

Fig. 1 Variation of neutron pairing energies (Δ_n) as function of mass number (A).

From these figures, it is clear that the magnitude of Δ_n and Δ_p decreases with mass number. To estimate the average pairing energies, we also introduced separate functional expression for $\bar{\Delta}_n = \frac{15.28}{A^{1/2}} MeV$ and $\bar{\Delta}_p = \frac{12.42}{A^{1/2}} MeV$ for even-even nuclides. Presently calculated pairing energies will be useful in the calculations of bandhead energies of two and multi-quasiparticle rotational bands.

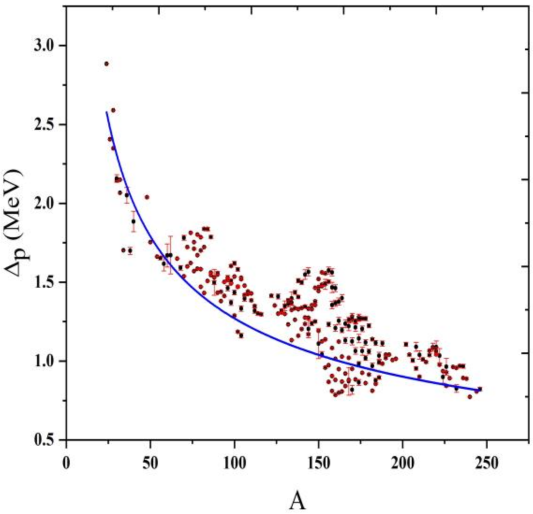

Fig. 2 Variation of proton pairing energies (Δ_p) as function of mass number (A).

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