Study of nuclear shapes of some even nuclei

*M. Singh[1], Y. Singh[2], Gulashan Mahajan[3], A. K. Varshney[4] and K. K. Gupta[5]

[1]Department of Physics, Greater Noida Institute of Technology, Greater Noida - 201306, INDIA
[2]Department of Physics, Govt. PG College, Dharamshala – 176215 (H.P.), India
[3]Department of Physics, Govt. College, Karsog – 175011 (H.P.), India
[4]Department of Physics, Govt. College, Palampur – 176061 (H.P.), India
[5]Department of Physics, Govt. PG College, Dhaliara – 177103 (H.P.), India
***singh.moti@gmail.com**

In Davydov asymmetric rotor (β – fixed, γ – fixed) the effect of (ΔK = 2) to state mixing is a classical illustration where γ and ground state levels of energy are plotted against the asymmetric defamation γ of the nucleus [1]. For γ close to 0^0, the Davydove model gives the symmetric rotor 'γ – band' energies. As γ increases the levels of γ – band decreases rapidly in energy and hence the even spin levels of the γ – band interact more and more with their partners in the ground band. The effect becomes extreme above γ = 20^0. The interaction faces a repulsion which is the origin of the upturn in E(4^+) near γ = 25^0. Thus, the γ – band levels from the couplets arranged as $(2_2^+, 3_1^+), (4_2^+, 5_1^+), 5_1^+$ ...

In another approach, say γ – unstable or Wilets – Jean model [2] the energies of γ – band are expected to form couplets arranged as $2_2^+, (3_1^+, 4_2^+), (5_1^+, 6_2^+)$ ... These two different couplets of γ – band energy levels are significant in distinguishing between γ – soft and γ – rigid shapes of a nucleus. Zamfir and Casten [3] introduced a term 'staggering indices' S(I) which has the form –

$$S(I) = \frac{S(I) + S(I-2) - 2S(I-1)}{E2_1^+} \qquad (1)$$

A clear distinction is arising in the γ – band in S (I) values, where both models exhibit in energy staggering, the sequencing is exactly opposite that is the phases of the S (I) in both the models would be reversed. Casten examined the values of the staggering indices obtained from the experimental data of even nuclei and found them to be matching with γ – soft predictions showing no evidence of γ – rigidity. Liao considered 140 even nuclei of mass region A = 64 – 200 where the most of the nuclei were found to be γ – soft but a few may be slightly triaxial. Almost all the axial nuclei are sightly γ – soft, some of them exhibiting shape transitions from axial to γ – soft to triaxial shape with increasing angular momentum [4]. In our view point since the nuclei possessing $15^0 < \gamma < 25^0$ are most appropriate to be considered in asymmetric rotor model description as they belong to transitional region. It will not be possible for a triaxial nucleus belonging to $\gamma \leq 20^0$ to show a zigzag pattern of S (I) versus spin (I) in theoretical values. We plotted a number of graphs in S (I) versus spin (I) for γ = 10^0, 15^0, 20^0 and 25^0 in asymmetric rotor model values [5]. Another thing associated with S (I) is nature of axial rotor. For an axial rotor model the energy spectra has the form –

$$E_I = AI(I+1) - BI^2(I+1)^2 \qquad (2)$$

Here S (I) are small and positive in magnitude that show no zigzag behavior, but increase slowly with increasing spin (I). Of course, S (I) = 0 for all spin (I) if B = 0. $E \propto I(I+1)$ are equally followed by axial as well as triaxial rotor. Thus, it becomes essential to distinguish triaxial rotors from axial rotors. This is done by corroborating them with the values of staggering indices S(I) in γ – band. We observe that the sign of S (I) changes alternatively for odd and even spins in the case of triaxial rotor but, S (I) in axial rotor does not change sign with spin. McCutchen referred to special solutions of the Bohr – Mottelson Hamiltonian that gave predictions for a triaxial structure in respect of five nuclei that is ^{112}Ru, ^{170}Er, ^{192}Os, ^{192}Pt and ^{232}Th [6].

In the present work the authors try to verify whether the nuclei proposed above except ^{170}Er, since it is discussed already in ref [7], are associated with triaxiality and if yes than to what extent. Attempts have been made to discuss ^{112}Ru in recent past but, in ref. 8 only γ – band is considered and not the γγ – band while we consider it to take essential γ – band as well as γγ – band together since both are generated simultaneously by rigid rotor of Davydov.

The asymmetry parameter γ is evaluated from the energy ratio of two band head energies ($R = E2_2^+/E2_1^+$) using the relation –

$$\frac{E2_2^+}{E2_1^+} = \frac{1+\left[1-\frac{8}{9}(\sin^2 3\gamma)\right]^{1/2}}{1-\left[1-\frac{8}{9}(\sin^2 3\gamma)\right]^{1/2}} \qquad (3)$$

This asymmetric parameter γ is fed to compute the rigid rotor model energies in γ and γγ – bands. The staggering indices S (I) for known experimental γ – band energies alongwith the rigid rotor energies are listed in table 1. The staggering indices for γγ – band in experiment and rigid rotor are listed in table 2.

Table – I

Staggering indices in γ – band for some even nuclei

S (I)/ Nucl.	Exp. Values*				ARM Values			
	^{112}Ru	^{192}Os	^{192}Pt	^{232}Th	^{112}Ru	^{192}Os	^{192}Pt	^{232}Th
S (4)	0.038	0.36	-0.21	0.09	0.56	0.476	0.44	0.02
S (5)	0.093	-0.17	0.001	0.05	-0.69	-0.52	-0.48	0.03
S (6)	0.339	-0.02	0.17	0.11	1.44	1.26	1.13	0.02
S (7)	-0.27	0.08	-0.07	0.05	-1.38	-1.62	-1.41	0.01
S (8)	0.64	-0.32	0.38	0.04	2.49	2.31	2.48	0.03
S (9)	-0.64			0.04	-2.83			0.03
S (10)	0.97			0.16	3.45			0.06
S (11)	-1.03			0.06	-3.77			-0.03
S (12)	1.37			0.19	4.39			0.105
S (13)	-1.50				-4.69			
S (14)	1.88				5.31			
S (15)	-2.05				-5.62			
S (16)	2.39				6.23			
S (17)	-2.51				-6.54			
S (18)	3.33				7.16			

Discussions on Tables:

Individual nucleus is discussed separately in the light of informations received from staggering indices.

1. · **^{112}Ru:** S (5) values in ARM and experiment are opposite in phase and then throughout till S (18). The phase of S (I) is similar at all the spins. A strong indication of γ – rigid nature with γ = 26.4^0.

2. **^{192}Os:** Phases of S (4) and S (5) are similar in experiment and ARM. S (6) and S (7) in experiment are negative and small, can be taken as nearly zero, and do not oppose the phases of ARM. S (7) in experiment is positive but small and again is taken as nearly zero and does not oppose the phase of S (7) in ARM. S (8) in experiment is comparatively large and negative in experiment but in ARM positive value is of considerable amplitude which is positive in sign and thus γ – soft nature of the nucleus is reflected at larger spin. The nucleus is γ – rigid in beginning and becoming γ – soft at energy of spin I = 8 with γ = 26^0

3. **^{192}Pt:** The values of S (4) reflect γ – soft nature that is positive in ARM and negative in experiment. S (5) is negative in ARM and in experiment it is positive but small and can be considered nearly zero so does not oppose the phase of S (5) in ARM. Later in S (6), S (7) and S (8), the phase tally in experiment and ARM both. Thus, the nucleus is γ – soft in beginning but later it becomes γ – rigid. The γ = 32.8^0 for this nucleus.

4. **^{232}Th:** The values of S (11) in ARM are – 0.03 and in experiment 0.06, both are nearly zero. All S (I) values are smaller and positive in experiment and ARM. The nucleus may be axial symmetric, these are the values which stand strongly in support of axial nature. The asymmetric parameter γ = 18.5^0 for ^{232}Th.

Table – II

Staggering indices in γ γ – band for some even nuclei

S (I)/Nucl.	Exp. Values*		ARM Values	
	^{112}Ru	^{192}Os	^{112}Ru	^{192}Os
S (6)	0.204	0.253	0.297	-0.02
S (7)	0.100	0.004	-0.127	0.08
S (8)	0.370		0.284	
S (9)	-0.253		-0.034	

***Experimental data have been taken from ref. 9.**

References:

1. A. S. Davydov and G. F. Filippov, Nucl. Phys. A 8, 237 (1958).
2. J. Wilets and M. Jean; Phys. Rev. 102, 788 (1956).
3. N. V. Zamfir et al; Phys. Lett. B 260, 265(1991).
4. Liao Ji – zhi; Phys. Rev. C 51, 141 (1995).
5. M. Singh et. al; Physics of Particles and Nuclei Letters, Vol. 19, No. 2, pp. 87–96 (2022)
6. E. A. McCutchen et al; Phys. Rev. C 76, 024306 (2007).
7. Proceedings of the DAE Symp. on Nucl. Phys. 64, 164 (2019)
8. M. Sugawara; Phys. Rev. C 99, 054323 (2019).
9. www.nndc.bnl.gov/nudat2/