Shape evolution of thorium-228, 230 and 232

Ummukulsu E* and Antony Joseph
[1] Department of Physics, University of Calicut, 673635, Kerala, INDIA

Introduction

Shape evolution of nuclei is one of the fundamental phenomena in nuclear physics. Theoretical study of shape evolution and coexistence of various shapes in a nucleus are useful for understanding the nuclear structure. The shape variation depends on the nuclear matter present in the nuclei [1]. The origin of nuclear shape and nuclear transition may be the evolution of shell structure with deformation, angular momentum and the number of valance nucleons. Closed shell nuclei, in general assume spherical shape whereas open shell nuclei exhibit deformed shape.

The most commonly observed shapes are spherical, prolate and oblate. Several theoretical approaches are available for understanding the shape evolution of nuclei [2–4]. Here, the energy density functional in terms of microscopic mean-field approach was used for the study of nuclear deformation and the shapes of thorium nuclei.

Theoretical Formalism

The microscopic mean-field theory gives information on the structure and shapes of nuclei. Mean-field theory is based on the energy density functional. Energy density functional is a functional of nucleon density matrices of single particle states. Here the meson exchange density functional is used for the calculations [5]. In the meson exchange model, the Lagrangian is defined in terms of the Lagrangian density

$$\mathcal{L} = \mathcal{L}_N + \mathcal{L}_m + \mathcal{L}_{int} \quad (1)$$

where $\mathcal{L}_N$ are the Lagrangian of free nucleons,

$$\mathcal{L}_N = \bar{\psi}(i\gamma_\mu \partial^\mu - m)\psi \quad (2)$$

*Electronic address: ummukulsu.e@uoc.ac.in

Here, $\mathcal{L}_m$ is the Lagrangian of the free meson field and the electromagnetic field given as,

$$\begin{aligned}\mathcal{L}_m = &\frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{2}m_\sigma^2\sigma^2 - \frac{1}{2}\Omega_{\mu\nu}\Omega^{\mu\nu} \\ &+ \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu - \frac{1}{4}\overrightarrow{R}_{\mu\nu}.\overrightarrow{R}^{\mu\nu} \\ &+ \frac{1}{2}m_\rho^2\overrightarrow{\rho}_\mu\overrightarrow{\rho}^\mu - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\end{aligned} \quad (3)$$

Also, here σ, ω and ρ represent the isoscalar-scalar meson, the isoscalar-vector meson and the isovector-vector meson, respectively. The corresponding masses are $m_\sigma, m_\omega, m_\rho$ and $\Omega_{\mu\nu}, \overrightarrow{R}_{\mu\nu}.F^{\mu\nu}$ are the field tensors. The interaction term L_{int} is,

$$\begin{aligned}L_{int} = &-g_\sigma\bar{\psi}\psi\sigma - g_\omega\bar{\psi}\gamma^\mu\psi\omega_\mu \\ &- g_\rho\bar{\psi}\overrightarrow{\tau}\gamma^\mu\psi\cdot\overrightarrow{\rho}_\mu - e\bar{\psi}\gamma^\mu\psi A_\mu\end{aligned} \quad (4)$$

with $g_\sigma, g_\omega, g_\rho$ and e being the coupling constants[5]. From this Lagrangian density, the Hamiltonian density is obtained by Legendre transformation. By integrating this Hamiltonian density, we obtain the total energy[6]. The quadrupole moments are expressed in terms of the deformation parameters a_{20} and a_{22} as,

$$Q_{20}^{(n,p)} = \frac{3A}{4\pi}R_0^2 a_{20}^{(n,p)} \quad (5)$$

and

$$Q_{22}^{(n,p)} = \frac{3A}{4\pi}R_0^2 a_{22}^{(n,p)} \quad (6)$$

The deformation parameters β and γ:

$\beta = \sqrt{a_{20}^2 + 2a_{22}^2}$ and $\gamma = arctan(\sqrt{\frac{a_{20}}{a_{20}}})$

The positive β values correspond to the $\gamma = 0^o$ axis (prolate shapes), and negative values to the $\gamma = 180^o$ axis (oblate shapes).

Results and Discussion

In the present study, we have plotted the axial and triaxial quadrupole binding energy surface of ^{228}Th, ^{230}Th and ^{232}Th. The results are obtained from the density dependent meson exchange model DD-ME2 energy density functional in relativistic mean-field theory and are displayed in Fig. 1. All energies in binding energy surface are normalized to the minimum energy. The triaxial binding energy surface is plotted in $\beta - \gamma$ plane with quadrupole deformation parameter β along the radial axis and γ along the angular axis. The color scale shown has energy in the unit of MeV.

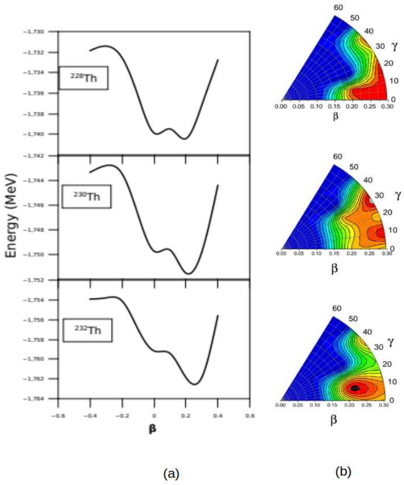

FIG. 1: *The axial quadrupole binding energy curve of ^{228}Th, ^{230}Th and ^{232}Th isotopes (a) and triaxial binding energy surface of ^{228}Th, ^{230}Th and ^{232}Th isotopes in $\beta - \gamma$ plane (b)*

The axial quadrupole binding energy curve gives the information on the shape of thorium nuclei. Fig. 1 (a) shows that the shape of thorium nuclei is prolate. In the case of ^{230}Th and ^{232}Th, well deformed prolate minima is observed and no well deformed minima in the case of ^{228}Th. Hence shape transition can be observed and well deformed minima is moving towards the prolate minima.

We also considered the role of triaxial deformation parameter γ in shape transition. The triaxial deformation binding energy surface of ^{228}Th, ^{230}Th and ^{232}Th are plotted in Fig. 1 (b). In the case of ^{228}Th, the location of minima is not observed due to flatness. This plot shows that ^{228}Th is prolate. ^{230}Th exhibits triaxial minima at $\gamma = 10^o$ and $\gamma = 30^o$. Hence ^{230}Th is triaxially deformed towards prolate. In the case of ^{232}Th, a well prounounced triaxial minima is observed around $\gamma = 10^o$. ^{232}Th is also a triaxially deformed prolate nucleus.

Conclusion

The microscopic mean-field theory gives information on the shape of nuclei. The axially deformed Relativistic-Hartree-Bogoliubov approach with the density dependent DD-ME2 interactions are used for the analysis. Triaxial quadrupole binding energy surface of ^{228}Th, ^{230}Th and ^{232}Th are plotted in the $\beta - \gamma$ plane. ^{228}Th, ^{230}Th and ^{232}Th exhibit prolate shape and ^{230}Th and ^{232}Th are triaxially deformed prolate nuclei. Nuclear shape variation may be due to the stretching of nuclear matter and moments.

Acknowledgments

Ummukulsu E acknowledges with thanks the financial support from CSIR, Govt. of India by way of the fellowship under SRF scheme.

References

[1] Z P Li et al., J. Phys. G: Nucl. Part. Phys. **43** 22, 024005 (2016)

[2] . Levit, Y. Alhassid, *Nucl. Phys. A*, 413 (1984).

[3] . Alhassid, S. Levit, and J. Zingman, *Phys. Rev. Lett.*,57, 1986.

[4] Bartel and P. Quentin, *Phys. Lett. B*, 152 (1985).

[5] T. Niksic, N. Paar *Comp. Phys. Com. doi.org/10.48550/arXiv.1403.4039* (2014)

[6] P.Ring, *Prog. Part. Nucl. Phys.* **37** 193(1996)