Two proton radioactivity of ^{38,39}Ti ,^{40}V and ^{42}Cr isotopes

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Introduction

Theorists have made significant efforts to predict two proton radioactivity and half-lives. There are numerous theoretical models that can explain two-proton radioactivity. The models that explain the two-proton radioactivity are the Effective liquid drop model (ELDM), the Generalized liquid drop model (GLDM), the Gamow-like model (GLM), the Skyrme-Hartree-Fock approach (SHF), the Unified fission model (UFM), and the study with screened electrostatic barrier (SEB). In addition to the theoretical models, there are many empirical formulas that calculate the radioactive logarithmic half-lives. I. Sreeja et al. proposed a four-parameter empirical formula for determining the logarithmic half-lives of two proton decays. Hong-Ming Liu et al. proposed a new two-parameter empirical formula based on the Geiger-Nuttall law for determining two proton logarithmic half-lives. Currently, only a few two-proton decays have been experimentally confirmed, while numerous theoretical predictions exist. In the present work, we have attempted to study the two-proton radioactivity of ^{38,39}Ti ,^{40}V ,^{42}Cr isotopes using the modified model of effective liquid drop model.

The Model

In the modified form of effective liquid drop model [1,2] the total interacting barrier potential energy is the sum of Coulomb potential, surface potential, and centrifugal potential energy. If Z_P, Z_D, and Z_{2p} are the atomic number of parent, daughter, and two proton fragment respectively. The total potential energy is constructed as,

$$V(\zeta)=\frac{KZ_D Z_{2P} e^2}{\zeta}+ V_s(\zeta)+ \frac{\hbar^2 l(l+1)}{2\mu\zeta^2} \qquad (1)$$

represents Coulomb potential energy, the second term shows surface potential and the last term is centrifugal energy. μ is the reduced mass of the 2p and daughter system, R_{2P}, ζ, and ξ are the geometrical parameters used to describe the dinuclear system.

$$R_P = r_0 {A_P}^{1/3} \qquad (2)$$

$$\bar{R}_i = \left(\frac{Z_i}{Z_P}\right)^{1/3} \qquad (i = 2p, D) \qquad (3)$$

A_P is the atomic mass number of parent r_0 is the nuclear radius constant. Z_P, Z_{2P} and Z_D are the atomic numbers of parent nucleus, daughter and 2p emitter nucleus respectively. R_D and R_{2P} are the radii of the daughter and the two proton respectively. ζ is the distance between the geometrical center of two nuclei. ξ is the distance between the center of the daughter and the plane of intersection of two nuclei. In this work on the 2p radioactivity we used the the constant mass asymmetry shape-Werner Wheeler (CMAS-WW) combination. The r_0 and λ_0 were determined as 1.12fm and 4.9610^{19}s^{-1}, respectively. The surface potential energy in terms of σ_{eff} (effective surface tension) is given by,

$$V_S=\sigma_{eff}(S_{2P}+S_D) \qquad (4)$$

From the above equation, S_D and S_{2p} are the surface area of the daughter and two proton nuclei. Q value for two proton decay is corrected by introducing the screening effect of the atomic electron. The surface area of the daughter and 2p fragment written in terms of the shape parameter

$$s_{2P} = 2\pi\bar{R}_{2p}\left(\bar{R}_{2p} + \zeta - \xi\right) \qquad (5)$$

$$s_D = 2\pi\bar{R}_D(\bar{R}_D + \xi) \qquad (6)$$

The barrier penetrability factor G is given as,

$$G = \exp\{\frac{-2}{\hbar}\int_{\zeta_0}^{\zeta_c}\sqrt{2\mu[V(\zeta) - Q]}d\zeta \qquad (7)$$

$\zeta 0$ and ζc are the inner turning point and outer turning point respectively .The decay rate is calculated by

$$\lambda=\lambda_0 G \qquad (8)$$

The half-life of two proton decay is given by

$$\tau_c= \ln 2/\lambda \qquad (9)$$

3. Result and Discussions

We have performed a detailed study on the ^{38,39}Ti ,^{40}V and ^{42}Cr isotopes and analyzed the probability of going one proton and two proton decay modes. Table 1 shows the computed Q values by introducing the screening effect of atomic electrons. For all isotopes two proton decay is energetically possible $Q_{2p}>0$. We computed the separation energy of one proton and two proton decays for these isotopes, which is shown in Table 1. Separation energy for one proton and two proton decays in terms of mass excess is computed using the equation is given below

$S_{(p)} = -\Delta M_{(A,Z)} + \Delta M_{(A-1,Z-1)} + \Delta M_H$

$S_{(2p)} = -\Delta M_{(A,Z)} + \Delta M_{(A-2,Z-2)} + 2\Delta M_H$

Where $\Delta M_{(A,\ Z)}$, ΔM_H, $\Delta M_{(A-1,\ Z-1)}$, $\Delta M_{(A-2,\ Z-2)}$ are mass excesses of the parent nucleus, proton and daughter nucleus respectively. Mass excess is taken from the mass table by Wang et al [3].

Parent Nucleus	Q_{2p}(MeV)	S(1p)(MeV)	S(2p) (MeV)
^{38}Ti	2.747	-0.061	-2.742
^{39}Ti	0.763	0.839	-0.7581
^{40}V	1.847	4.608	-1.842
^{42}Cr	1.007	0.879	-1.002

Table 1: The separation energies of one proton and two protons and the Q values of two proton radioactivity for various nuclei

The isotopes of Titanium $^{38,\ 39}$Ti have a lower S(2p) value, and both have a larger possibility of showing two protons decaying than one proton decaying. In ^{40}V, ^{42}Cr nuclei two proton separation energy is lower than one proton separation energy. These findings suggest that these nuclei have a high chance of showing two proton radioactivity as compared to one proton decay.

Driving potential represents the height of barrier potential. The lower the driving potential which makes the barrier tunnelling process easier. To find the fragment with the lowest driving potential, we calculated the driving potential of all possible fragments (1p, 2p, ^{3}He, ^{4}He, ^{4}Li, ^{5}Li, ^{6}Li, etc.). In figure 1, the isotopes ^{38,39}Ti have a minimum driving potential for two protons with an angular momentum of l = 0. Similar results are obtained in the cases of ^{40}V and ^{42}Cr isotopes.

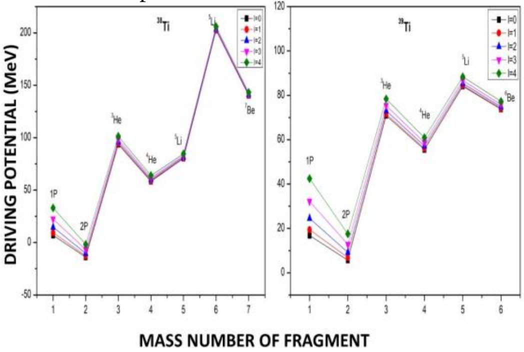

Fig. 1 The driving potential for ^{38}Ti and ^{39}Ti nuclei as a function mass number of the fragment for angular momentum (l=0,1,2,3,4)

When the angular momentum is increased, the fluctuation in driving potential follows a pattern identical to that of the ground state. In comparison to other fragments, the minimum driving potential shows that the height of the barrier potential is low for two proton and that tunnels easily through the barrier potential.

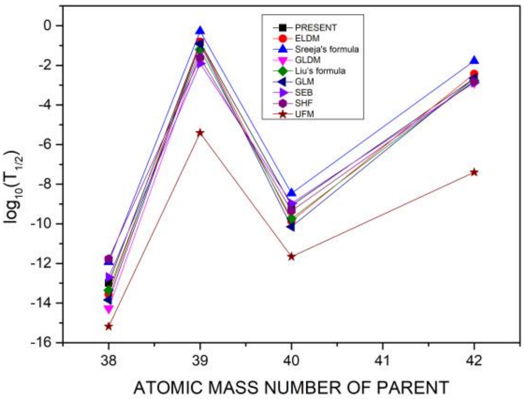

Fig. 2 Computed half-life is compared with the other six theoretical model predictions and also with two formula predictions.

From Figure 2, it is clear that the computed half-life agrees well with the other theoretical model predictions and formula predictions

References

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