# Theoretical investigation of double $\alpha$-emission

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Poenaru and Ivaşcu proposed the possible existence of double $\alpha$ radioactivity in 1980s [1]. Recently, Tretyak investigated the decay process in almost eighty nuclides to explore the possibile decays and the experimental limit on the half-life is set to be greater than $10^{20}$ years as obtained from the study of $^{209}$Bi nucleus[2]. We consider the quantum tunneling of already formed two $\alpha$-particles through the interaction potential barrier $V(r,\theta)$ consisting of the Coulomb potential $V_C(r,\theta)$ [3],

$$V_C(r,\theta) = \frac{2Z_d e^2}{r}(1 - \mathrm{e}^{-S}) \tag{1}$$

with $S$ is a function as described below:

$$S = \phi(\theta)r + 0.5(\phi(\theta)r)^2 + 0.35(\phi(\theta)r)^3 \tag{2}$$

and, the proximity potential $V_P(r,\theta)$ [4],

$$\begin{aligned} V_P(r,\theta) &= 4\pi\gamma \frac{C_{2\alpha}C_d}{C_{2\alpha}+C_d} \times \\ & \left(\frac{-0.0527}{0.025e^{0.12g} + 0.0085e^{1.52g'}}\right. \\ &+ \left.\frac{1.956}{r^2} - \frac{0.142}{r}\right) \end{aligned} \tag{3}$$

where $\gamma = 0.9517\sqrt{1 - 2.6\left(\frac{A_d - 2Z_d}{A_d}\right)^2}$,
and $g' = r - C_{2\alpha} - C_d$.

"$\theta$" is the orientation angle of the emitted $\alpha$-particles w.r.t. symmetric axis of deformed daughter nucleus and, the nuclear radius $R_i$ with $i = \alpha, d, p$ can be found as:

$$\begin{aligned} R_i = \{1.28A_i^{1/3} - 0.76 + 0.8A_i^{-1/3}\} \times \\ (1 + \beta_2 Y_{20}(\theta) + \beta_4 Y_{40}(\theta)). \end{aligned} \tag{4}$$

$\beta_2$ and $\beta_4$ are the quadrupole and hexadecapole deformations of nuclear ground states. Also, $C_i = R_i - \frac{b^2}{2R_i}$ with $b$ as surface width chosen to be 1 fm. Now, the half-life of the nucleus can be found as,

$$T_{1/2}^{2\alpha,th}(s) = \frac{\ln 2}{K v_0 P_0}. \tag{5}$$

Here, $v_0 = \frac{2E_v}{h}$ is the assault frequency while $E_v$ is the vibrational energy such that [5]:

$$E_v = Q_{2\alpha}\left(0.056 + 0.039\exp\left(\frac{4 - A_{2\alpha}}{2.5}\right)\right) \tag{6}$$

The tunneling probability $K$ through the potential barrier can be evaluated as [6]:

$$K = \int_0^{\pi/2} \sin\theta \; T(\theta) \; d\theta \tag{7}$$

where $T(\theta)$ is the transmission coefficient [7]:

$$T(\theta) = \frac{1}{1 + \exp(q(\theta))} \tag{8}$$

and

$$q(\theta) = \left[\frac{2}{\hbar}\int_{a'(\theta)}^{b'(\theta)} \sqrt{2\mu\, m_n(V(r,\theta) - Q_{2\alpha})}dr\right] \tag{9}$$

where $m_n$ is mass of nucleon. The turning point $b'(\theta)$ is fixed such that $V(b'(\theta)) = Q_{2\alpha}$ while the turning point $a'(\theta)$ is taken as $R_{2\alpha} + R_d$. The preformation probability $P_0$ is [8],

$$\log_{10} P_0 = h' + f'Q_{2\alpha} + gQ_{2\alpha}^2 \tag{10}$$

where $f' = -0.25736$, $g = 6.37291 \times 10^{-4}$ and $h' = 3.35106$. The disintegration energies $Q_{2\alpha}$ are calculated using binding energies as reported in [9]. We have calculated the logarithm of half-lives of medium and heavy mass nuclei which can decay with the emission of two $\alpha$-particles and the obtained results are plotted with respect to $\frac{Z_d}{\sqrt{Q_{2\alpha}}}$ in the Figure 1.

| Parent Nucleus | Daughter Nucleus | $Q_{2\alpha}$ | $\log_{10}[T^{2\alpha}_{1/2}(\text{yr})]$ | | | | |
|---|---|---|---|---|---|---|---|
| | | | Theor. | CPPM | MGLDM | Q | Ref. [2] |
| | | (MeV) | | | | | |
| $^{148}$Sm | $^{140}$Ce | 3.933 | 52.848 | 59.40 | 57.77 | 55.41 | 58.11 |
| $^{152}$Gd | $^{144}$Nd | 4.238 | 51.424 | 57.77 | 56.28 | 54.00 | 56.61 |
| $^{156}$Dy | $^{148}$Sm | 4.006 | 57.183 | 62.75 | 60.77 | 58.43 | 64.28 |
| $^{174}$Hf | $^{166}$Er | 4.288 | 61.919 | 68.63 | 65.84 | 63.57 | 69.91 |
| $^{180}$W | $^{172}$Yb | 4.828 | 56.668 | 64.04 | 60.68 | 58.55 | 64.15 |
| $^{184}$Os | $^{176}$Hf | 5.535 | 50.331 | 57.96 | 54.29 | 52.33 | 57.15 |
| $^{186}$Os | $^{178}$Hf | 4.646 | 63.279 | 77.58 | 73.41 | 71.15 | 75.90 |
| $^{190}$Pt | $^{182}$W | 6.153 | 46.31 | 54.33 | 50.33 | 48.53 | 52.43 |
| $^{192}$Pt | $^{184}$W | 4.630 | 67.338 | 77.30 | 72.71 | 70.52 | 74.15 |
| $^{196}$Hg | $^{188}$Os | 4.527 | 72.867 | 83.69 | 78.46 | 76.25 | 79.53 |
| $^{232}$Th | $^{224}$Rn | 8.230 | 40.398 | 45.62 | 42.59 | 41.30 | 46.80 |
| $^{234}$U | $^{226}$Ra | 9.709 | 31.231 | 35.72 | 32.86 | 31.92 | 37.80 |
| $^{238}$U | $^{230}$Ra | 8.022 | 43.292 | 48.73 | 45.53 | 44.19 | 50.97 |

TABLE I: Results for the logarithmic values of half-lives of medium and heavy nuclei obtained using eq. (5) and other theoretical methods.

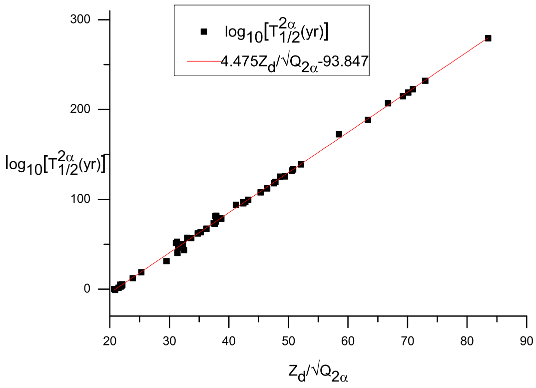

FIG. 1: Plot of the logarithmic values of half-lives w.r.t. $\frac{Z_d}{\sqrt{(Q_{2\alpha})}}$ for medium and heavy nuclei.

A very good linear fit is obtained as well. We have also compared our results for few cases with those predicted using other theoretical methods [10] viz. CPPM, MGLDM, the model using disintegration dependent form of preformation probability (Q) and the results are reported in [2] (see Table I).

## References

[1] D. Poenaru and M. Ivaşcu, Journal de Physique Lettres **46**, 591, (1985).
[2] V. I. Tretyak, Nucl. Phys. At. Energy **22**, 121 (2021).
[3] D. Brink and N. Takigawa, Nucl. Phys. A **279**, 159 (1977).
[4] G. Royer, Nucl. Phys. A **848**, 279 (2010).
[5] D. N. Poenaru *et al.* Phys. Rev. C **32**, 572 (1985).
[6] Y. Qian *et al.* Phys. Rev. C 83, 044317 (2011).
[7] L. D. Landau and E. Lifshitz, Quantum Mechanics, (Pergamon Press, 1977).
[8] K. P. Santhosh *et al.* Phys. Rev. C **97**, 064616 (2018).
[9] https://www.nndc.bnl.gov/nudat2/.
[10] K. P. Santhosh and T. A. Jose, Phys. Rev. C **104**, 064604 (2021).

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