Nuclear asymmetry energy within semiclassical approach

Robinjeet Singh[1],* Inderjit Singh[1], Deepika Pathak[1], Harjeet Kaur[1], and Sudhir R. Jain[2]
[1] *Department of Physics, Guru Nanak Dev University, Amritsar-143005. and*
[2] *Nuclear Physics Division, Bhabha Atomic Research Centre, Mumbai-400085.*

1. Introduction

The nuclear energy given in the Bethe-Weizsäcker formula can be written as

$$E(A,Z) = -c_1A + c_2A^{2/3} + a_a(A)\frac{(N-Z)^2}{A} + \text{shell and pairing corrections} \quad (1)$$

The first three terms in the R.H.S. of above eq. correspond to the volume, surface and symmetry components of energy, respectively. Asymmetry energy including volume and surface contributions is expressed as [1]

$$\frac{1}{a_a(A)} = \frac{1}{a_A^V} + \frac{A^{-1/3}}{a_A^S} \quad (2)$$

$$\text{where } a_A^V = S\left(1 + A^{-1/3}\kappa\right) \quad (3)$$

$$a_A^S = \frac{S}{\kappa}\left(1 + A^{-1/3}\kappa\right) \quad (4)$$

are volume and surface coefficients. The study of nuclear symmetry energy (NSE) "S" and its dependence on temperature is getting attention as it is a crucial parameter in exploring the properties of the atomic nucleus and the neutron star.

In the CDFM (Coherent density fluctuation model), S for finite nuclei is [2]

$$S = \int_0^\infty |\mathcal{F}(x)|^2 \, S^{ANM}(x)\, dx \quad (5)$$

where the symmetry energy $S^{ANM}(x)$ for asymmeteric nuclear matter (ANM) with density $\rho_0(x)$ has the form [2]

$$S^{ANM}(x) = 41.7\rho_0^{2/3}(x) + 148.26\rho_0(x) + 372.84\rho_0^{4/3}(x) - 769.57\rho_0^{5/3}(x) \quad (6)$$

and $|\mathcal{F}(x)|^2 = -\frac{1}{\rho_0(x)}\left(\frac{d\rho_{\text{total}}(r)}{dr}\right)\Big|_{r=x}$ (7)

with the normalization $\int_0^\infty |\mathcal{F}(x)|^2 dx = 1$

and $\rho_{\text{total}}(r) = \rho_p(r) + \rho_n(r)$.

$\rho_p(r)$ *and* $\rho_n(r)$ being the proton and neutron densities respectively [3]. The weight function $|\mathcal{F}(x)|^2$ in Eq. (7) for monotonically decreasing local density is described by the 3-D semiclassical density $\rho(r,\theta)$ upto $\mathcal{O}(\hbar^2)$ as [4] :

$$\rho(r,\theta) = \frac{1}{3\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}(\mu - V(r,\theta))^{3/2} - \left(\frac{1}{24\pi^2}\left(\frac{2m}{\hbar^2}\right)^{1/2}\left[\frac{\nabla^2 V(r,\theta)}{(\mu - V(r,\theta))^{1/2}} + \frac{(\nabla V(r,\theta))^2}{4\,(\mu - V(r,\theta))^{3/2}}\right]\right) \quad (8)$$

where

$$V(r,\theta) = \frac{m\omega^2 r^2}{2} - \beta_2 m\omega^2 r^2 Y_{20}(\theta,\phi)$$

is the potential energy for 3-D harmonic oscillator. Moreover, the radial local density can be obtained as [3]:

$$\rho(r) = \int_0^\pi \rho(r,\theta)((\sin(\theta))/2 \;\; d\theta. \quad (9)$$

For our calculations, we fixed the chemical potential μ for neutron and proton by following relation [4]

$$N, Z = \int_0^{\mu_{n,p}} g_{n,p}(E)\; dE \quad (10)$$

and $g_{n,p}(E)$ is the single particle level density for anisotropic harmonic oscillator potential

*Electronic address: robinbhoma18@gmail.com

[5] which is obtained by analytical continuation. The ratio of volume to surface energy coefficients using symmetric energy dependence on density $S(\rho)$ is given as [1]

$$\kappa = \frac{a_A^V}{a_A^S} = \frac{3}{r_0\rho_0}\int_0^\infty \left[|\mathcal{F}(x)|^2 x\rho_0(x) \left(\frac{S(\rho_0)}{S(\rho_0(x))} - 1\right)\right]dx \quad (11)$$

$$S(\rho_0) = S^{ANM}(\rho_0)\ ,\ S[\rho_0(x)] = S^{ANM}(\rho_0(x))$$

where ρ_0 is the nuclear matter equilibrium density and $r_0 = 1.15fm$ is the radius of the nuclear volume per nucleus given as

$$\rho_0 = \frac{3}{4\pi r_0^3}\ \text{fm}^{-3},\ \rho_0(r) = \frac{3A}{4\pi r^3}\ \text{fm}^{-3}$$
$$r = r_0 A^{1/3} fm$$

2. Results and Discussion

The numerical results for symmetry energy, its volume and surface coefficients and their ratio κ are reported Table I. Our results show that the values of the ratio κ for even-even Ni-isotopes (A=70-84) lie in the range $1.46 \leq \kappa \leq 1.83$. Relatively large values of κ are obtained for deformed nuclei as compared to the spherical ones. Moreover, a_A^V takes the values ranging from 27.325 to 39.205 while a_A^S is found to be varying from 17.5 to 21.3 for the nuclei under consideration.

Subsequently, using the values of κ and the volume coefficient a_A^V, the neutron skin thickness is calculated as the difference of the r.m.s. radii of neutrons and protons given by [7]:

$$\Delta R = (0.77\ r_0\ A^{1/3}) \frac{A}{6\ N\left(1 + \frac{A^{2/3}}{\kappa}\right)} \left[\frac{N-Z}{Z} - \frac{0.7103\ A^{2/3}}{28\ a_A^V}\left(\frac{10}{3} + \frac{A^{1/3}}{\kappa}\right)\right]. \quad (12)$$

A linear dependence of neutron skin thickness on mass number is observed in fig. 1 and an anomalous behaviour is observed at $A = 70$ and 74 which belong to deformed isotopes of Ni.

A	β_2	S	κ	a_A^V	a_A^S	ΔR
		(MeV)		(MeV)	(MeV)	(fm)
70	0.179	27.134	1.833	39.205	21.384	0.043
72	0.000	21.459	1.469	29.035	19.770	0.038
74	0.140	24.331	1.817	34.859	19.189	0.053
76	0.000	20.998	1.500	28.432	18.959	0.047
78	0.000	20.779	1.514	28.143	18.584	0.052
80	0.000	20.593	1.533	27.919	18.215	0.056
82	0.000	20.383	1.546	27.638	17.873	0.061
84	0.000	20.162	1.556	27.325	17.562	0.065

TABLE I: Deformation parameter [8] and the results for asymmetry energy, its volume and surface coefficients, their ratio $\kappa$and neutron skin thickness for Ni-isotopes.

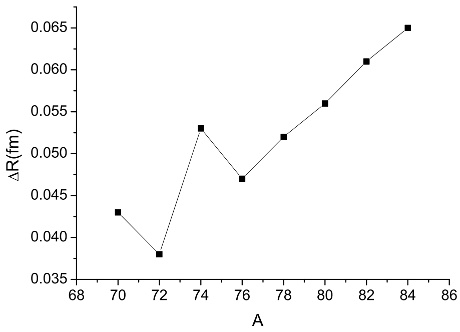

FIG. 1: Neutron skin thickness plotted w.r.t. A for Ni-isotopes.

References

[1] M.K. Gaidarov et al., Phys. Rev. C **94**, 014319 (2016).
[2] M.K. Gaidarov et al., Phys. Rev. C **85**, 064319 (2012).
[3] Q. Shou, Y. Ma, P. Sorensen et al. Phys.Lett. B **749**, 215 (2015).
[4] M. Brack and R.K. Bhaduri, Semiclassical Physics. Addison-Wesley (1997).
[5] M. Brack and S. R. Jain, Phys. Rev. A **51**, 3462 (1995).
[6] I. Ragnarsson, S.G. Nilsson, Shapes and Shells in Nuclear Structure (1995).
[7] P. Danielewicz, Nuclear Physics A **727**, 233 (2003)
[8] https://www.nndc.bnl.gov/