Perspectives of Extrapolation of Atomic Masses of Nuclides far from the Valley of Stability

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Introduction:

In recent years, our knowledge on atomic masses far from the valley of stability has been increased due to the advent of new measuring techniques. Application of these masses, e.g. for astrophysical calculations, requires data for even far more unstable isotopes which are generally derived from global mass models. In the absence of experimental data the values of the unknown nuclear masses are obtained through theoretical predictions but, unfortunately, there is a lack of consensus and most predictions differ drastically from each other, especially in the region of large neutron excess [1,2]. Local interpolations or extrapolations of the experimental mass values have been tried as an alternative and more plausible way to reach the nuclides far from the valley of stability.

Methods:

***i) Isobaric Multiplet Mass Equation*: ** The Isobaric Multiplet Mass Equation (IMME) is a powerful interpolation/extrapolation method for lighter nuclei. Mass values derived from IMME had been included in older mass evaluations and the recent high-precision mass measurements have led to a renewed interest in this technique [3].

***ii)Way-Wood Diagrams*:** Way and Wood [4] derived graphical presentations of nuclear decay data on the basis of the semi-empirical Bethe-Weizsacker [5,6] mass formula. Under the assumption that the atomic masses are smoothly varying functions of N and Z (as the semi empirical mass formula) the lines were extrapolated to unmeasured nuclei.

***iii)*Garvey and Kelson Relations:** In 1966 Garvey and Kelson presented a relation based on an independent particle model of the nucleus [7] :
$M(N+2, Z-2)-M(N, Z)+M(N, Z-1)-M(N+1, Z-2)+M(N+1, Z)-M(N+2, Z-1) = 0$ This equation relates six nuclides. With five known masses, a sixth unknown one can be predicted. These global mass formulas, however, will in general be too approximate to make predictions to a sufficient degree of accuracy.

***iv)Interactive graphical program*:** Audi-Wapstra successfully employed a local extrapolation method [8] based on the systematics and smoothness of the mass surface and its derivatives. They used an interactive graphical program to make estimates for unknown masses from the trends in the mass surfaces. However, this procedure encompasses a "subjective" component in the form of individual judgments.

***v)Weighted Slope Method*:** A point-to-point local extrapolation to systematically take into account the local variation in the trends that are observed in the derivative sheets. The local trend was quantified in terms of the value of the slopes connecting one isotope to the next. The extrapolated slope for the prediction was obtained by averaging the slopes of three of the immediate neighbors. A weighted average of the three slopes is used, with higher weightage placed on the slope closest to the one to be found. The extrapolated slope value was used to find the unknown value of the derivative. Since this extrapolation method is based on systematic local extrapolation, the method is very effective in following local variations in trends present in the derivative sheets.

Results and Discussion:

A humble attempt is made in Table 1 where we have considered a typical region of As, Se, Br, Kr, Rb, Sr and Y and compared our predictions [9] with two available extrapolations and one of the most recent mass models WS4 [10], we could see the deviations are well within the uncertainties demonstrating the usefulness of the weighted slope method.

Table 1: Comparison of the present extrapolated mass excess values with the corresponding values from two other extrapolation methods and the most recent theoretical mass model values.

				Mass Excess (keV)				**Deviations (keV)**		
A	**Z**	**N**	**Nuclide**	**WS4[10]**	**P.W.[9]**	**AME16[8]**	**G.K[7]**	Δ_{PW-WS4}	Δ_{PW-AME}	$\Delta_{PW-G.K}$
84	31	53	Ga	-43930	-44034 100	-44094	-44069	104	-60	-35
85	31	54	Ga	-39732	-39832 30	-39744	-39908	100	88	-76
86	31	55	Ga	-33392	-33721 400	-34080	-33961	329	-359	-240
87	31	56	Ga	-28314	-28836 500	-29250	-29345	522	-414	-509
87	32	55	Ge	-43272	-43826 150	-44080	-43832	554	-254	-6
88	32	56	Ge	-40055	-40310 300	-40140	-40021	255	170	289
89	32	57	Ge	-33557	-33531 500	-33730	-33526	-26	-199	5
90	32	58	Ge	-28658	-29496 600	-29220	-29037	838	276	459
88	33	55	As	-50428	-50514 50	-50720	-50626	86	-206	-112
89	33	56	As	-46473	-46556 70	-46800	-46816	83	-244	-260
90	33	57	As	-40718	-40938 250	-41330	-40955	220	-392	-17
91	33	58	As	-36173	-37047 300	-36900	-36763	874	147	284
92	33	59	As	-29818	-30743 400	-30980	-30739	925	-237	4
91	34	57	Se	-50307	-50426 140	-50580	-50238	119	-154	188
92	34	58	Se	-46620	-46573 170	-46720	-46734	-47	-147	-161
93	34	59	Se	-40542	-40570 280	-40720	-40951	28	-159	-381
94	34	60	Se	-36304	-36971 350	-36800	-37104	667	171	-133
95	34	61	Se	-29803	-29589 450	-30460	-30761	-214		-1172
94	35	59	Br	-47686	-47543 150	-46800	-47662	-143	743	-119
95	35	60	Br	-43846	-43798 250	-43770	-43934	-48	28	-136
96	35	61	Br	-38132	-38621 270	-38160	-38240	489	461	381
97	35	62	Br	-33527	-33616 400	-34060	-33386	89	-444	230
98	35	63	Br	-28879	-28590 500	-28250	-27987	-289	340	603
98	36	62	Kr	-44013	-44588 200	-44310	-44025	575	278	563
99	36	63	Kr	-38132	-38948 300	-38760	-38913	816	188	35
100	36	64	Kr	-35470	-35730 300	-35050	-34921	260	680	809
100	37	63	Rb	-46143	-46150 150	-46290	-46524	7	-140	-374
101	37	64	Rb	-42445	-42920 190	-42558	-42948	475	362	-28
102	37	65	Rb	-36852	-37830 250	-37253	-37402	978	577	428
103	37	66	Rb	-32348	-32983 350	-33610	-32348	635	627	635
103	38	65	Sr	-47053	-47308 120	-47420	-47299	255	-112	9
104	39	65	Y	-53670	-54047 70	-54060	-54021	377	-13	26

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