Structure study of the exotic ^{72}Ca dripline nucleus

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Introduction

Recently, nuclear physicists have focused a lot of their attention on the nuclear landscape, especially the dripline region. There are nuclei with excess neutrons or excess protons on either side of the nuclear stability line. In the plot of Z (number of protons) vs. N (number of neutrons), a point will occur when no additional neutrons can be added to form a stable nucleus for a given Z. If we locate other such locations for different Z values, we will obtain a line linking those points that is referred to as a drip line. The discovery of halo nuclei are the most notable discovery near these driplines. For the first time following the development of radioactive ion beam (RIB) facilities of RIKEN, Tanihata, and his colleagues are credited with the first-ever identification of the two-neutron halo structure in neutron-rich ^{11}Li in 1985[1]. The following are further typical instances of two-neutron halo nuclei: ^{6}He, ^{11}Li, ^{14}Be, ^{18}C, ^{20}C, ^{22}C, ^{40}Mg, ^{42}Mg, ^{44}Mg, and so on. This intriguing nuclear structure consists of a dense stable core surrounded by a low-density envelope with a lengthy extension.
According to literature studies, ^{72}Ca is one of the most exotic neutron-rich nucleus that is currently available for theoretical and experimental research. S.Q. Zhang et al., in 2002, used RCHB theory to predict two neutron separation energies of around 0.04 MeV, and they proposed that A > 60 exotic nuclei are all weakly bound and can be easily scattered to the continuum spectra due to pair interaction [2]. In 2004, R. I. Betan et al. studied ^{72}Ca using a shell model in the complex energy plane (CXSM) and discovered that the ground state of ^{72}Ca is located at -295 KeV as well as a low-lying two-particle resonance with an energy of $(0.550, -0.350)$ MeV[3]. In 2022, W. Horiuchi et al. proposed that the rms radius of ^{72}Ca is extremely large because of the low binding energy of -0.03 MeV. They did this by describing the weakly bound correlated neutron motion around a heavy core using a forbidden state free locally peaked Gaussian expansion method[4].
In the current work, we have used a sophisticated theoretical framework to investigate the ground and resonance states of ^{72}Ca in the context of the few-body model. We consider a three-body model of ^{72}Ca with a structureless ^{70}Ca core and two valence neutron(s). Using the standard GPT nucleon-nucleon potential and standard SBB core-nucleon potential, we first determine the three-body system's bound state. The ^{71}Ca subsystem is totally unbound, which led to the choice of the core-nucleon potential parameters. The ground state wave function was then used to build the one-parameter family of isospectral potentials. The parameter is set to provide three distinct barriers: a sharp barrier at energy $E(> 0)$, a positive barrier that aids in particle trapping inside the deep well, and deep well. With a peak at resonance energy, the likelihood of the particle being stuck inside the well-barrier combination is computed for different positive energies. After determining the resonance energy, we used the WKB approximation to get the resonance width.

Method

The Hyperspherical Harmonic Expansion Method is used to solve an uncommon three-body ^{72}Ca nuclear system. Two circling valence nucleons are represented as particles " j ", and " k," respectively, whereas the much heavier nuclear core ^{70}Ca is designated as particle " i ". In the i^{th} partition, where i, j, and k are all cyclic, the Jacobi coordinates are defined as follows: Khan et al. (2021) [5] as having recently tested the current technique for the core plus two-neutron three-body model with success.

$$\vec{x_i} = a_i(\vec{d_j} - \vec{d_k}) \tag{1}$$

$$\vec{y_i} = \frac{1}{a_i}\left(\vec{d_i} - \frac{m_j\vec{d_j} + m_k\vec{d_k}}{m_j + m_k}\right) \tag{2}$$

$$\vec{R} = \frac{\left(m_i\vec{d_i} + m_j\vec{d_j} + m_k\vec{d_k}\right)}{M} \tag{3}$$

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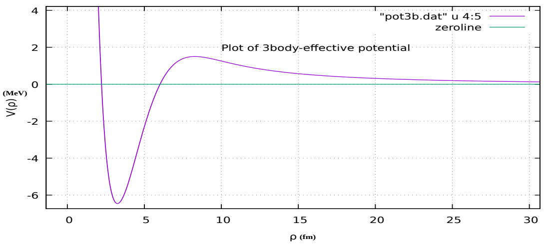
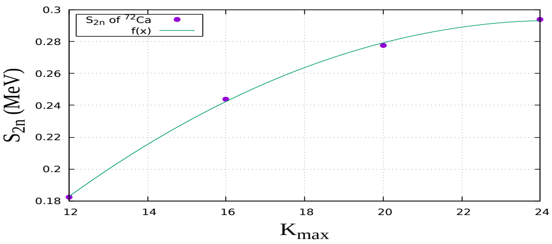
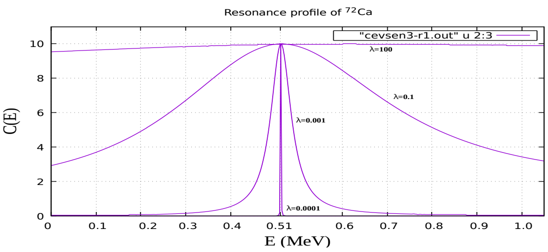


(a) Representation of three-body effective potential of ^{72}Ca.



(b) Representation of two-neutron separation energy with K_{Max} of ^{72}Ca



(c) Representation of resonant state of ^{72}Ca.

where a_i is const. ; $m_i, \vec{d_i}$ are the mass and position of the i^{th} particle and $M = m_i + m_j + m_k, \vec{R}$ is the centre of mass (CM) of the system. The Global radius ρ, an invariant under three dimensional rotations and permutations of particle indices together with the five angular variables $\Omega_i \rightarrow \{\phi_i, \theta_{x_i}, \phi_{x_i}, \theta_{y_i}, \phi_{y_i}\}$ constitute hyperspherical variables of the system. It should be noted that hyper 'angles Ωi are determined by the partition "i" chosen. The Schrödinger equation is rewritten in terms of hyperspherical variables (ρ, Ω_i):

$$\left[-\frac{\hbar^2}{2\mu}\{\frac{1}{\rho^5}\frac{\partial}{\partial\rho}(\rho^5\frac{\partial}{\partial\rho}) - \frac{\hat{\mathcal{K}}^2(\Omega_i)}{\rho^2}\right] + [V(\rho,\Omega_i) - E]\,\Psi(\rho,\Omega_i) = 0 \quad (4)$$

where $V(\rho, \Omega_i)$ is the total interaction potential in the partition "i", and the eigenvalue equation is satisfied by the square of the hyper angular momentum operator $\hat{\mathcal{K}}^2(\Omega_i)$.

TABLE I: **The calculated results for ^{72}Ca a and other reference sources accessible in the literature.**

K_{Max}	S_{2n} (MeV)	$P_{lx=0}$	$E_{lx=0}$(MeV)
12	0.1643	0.8333	-0.1817
16	0.2240	0.8344	-0.2313
20	0.2568	0.8352	-0.2587
24	0.2723	0.8354	-0.2712
...			
∞	0.3022		
State	Observables	Present work S_{2n}	Others work
0^+	S_{2n} (MeV)	0.3022	-0.295, 0.04[2, 3]
0_2^+	E_R(MeV) Γ(MeV)	0.51 MeV 0.678 MeV	0.550, -0.350[3] -

Results and Discussions

In this research, we investigated resonances in nuclei with at least one bound state as well as those without a bound ground state using a unique method. For this reason, the Hyperspherical Harmonic Expansion Method (HHEM) for few-body nuclear systems (like ^{72}Ca) is used in conjunction with the Super Symmetric Quantum Mechanics (SSQM) formalization. It is very challenging to solve the few-body Schrödinger equation for weakly bound exotic systems in a way that leads to a fully converged solution. On the other hand, this approach may be very beneficial for researching weakly bound states, continuum-bound states, and resonances. The method presented here is a robust one and can be applied to any weakly bound system in which the binding energies steadily converge as K_{max} values rise. Table 1 summarises the results and shows ^{72}Ca as a potential candidate for a 2-neutron halo.

Acknowledgments

The Authors would like to express an appreciation of necessary facilities from Aliah University.

References

[1] I. Tanihata et al., Phys. Rev. Lett. **55**, 2676 (1985).
[2] S. Q. Zhang et al., Chin. Phys. Lett. **19**, 312 (2002).
[3] R. I. Betan et al., Phys. Lett. B. **584**, 48 (2004).
[4] W. Horiuchi et al., Phys. Rev. C. **105**, 024310 (2022).
[5] Md. A. Khan, M. Hasan, S. H. Mondal, M. Alam, Nucl. Phys. A **1015**,122316 (2021).