Reconstruction of nuclear matter parameters in a Bayesian approach

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The posterior distributions of nuclear matter properties are reconstructed using a Bayesian technique from the EoS of neutron star matter. Appropriate prior distributions are chosen to put constraints on lower-order parameters as imposed by the finite nuclei observables.The calculations are performed on two sets of pseudo data on the EoS whose real models are known.The accompanying uncertainties are also greater and the median values of second or higher order NMPs exhibit considerable deviation from their true values. The sources of these inherent uncertainties are discussed.

Introduction

The equation of state (EoS) of dense matter may be constrained in a narrow window by the bulk properties of neutron stars(NSs).EoS of NS matter can be decomposed in terms of two main components as follows

$$\varepsilon(\rho, \delta) \;=\; \varepsilon(\rho, 0) + J(\rho)\delta^2 + ..., \qquad (1)$$

where, $\varepsilon(\rho, 0)$ and $J(\rho)$ are the EoS for symmetric nuclear matter and symmetry energy, respectively and $\delta = \left(\frac{\rho_n - \rho_p}{\rho}\right)$ is the isospin asymmetry parameter with ρ_n and ρ_p being the neutron and proton densities, respectively. $\varepsilon(\rho, 0)$ and $J(\rho)$ can be further expanded in terms of various nuclear matter parameters(NMPs) which are e_0 (binding energy), K_0 (incompressibility),Q_0 (skewness), Z_0 (kurtosis) and J_0 (symmetry energy at saturation),L_0 (slope of symmetry energy) $K_{sym,0}$ (curvature), $Q_{sym,0}$, $Z_{sym,0}$, respectively (see for details [1]). It is usually believed that the NMPs may be reconstructed from the knowledge of EoS for the NS matter. In the present work we explore the possibility of reconstruction of NMPs from the accurate pseudo-data from the EoS of NS matter obtained from Taylor and n/3 expansions referred hereafter as E1 and E2, respectively. We use a Bayesian technique to reconstruct the marginalised posterior distributions(PDs) for the NMPs using EoS for the NS matter. The pseudo data for the EoS of NS matter are constructed using a suitable set NMPs (see Table I) which satisfies i) Thermodynamic stability ii)$J(\rho) \geq 0$ iii)Causality iv)The EoS should give maximum mass of neutron star greater than $2M_\odot$ (see for details [1]).

TABLE I: The values of nuclear matter parameters (in MeV) which are employed to construct various pseudo data using the Taylor and $\frac{n}{3}$ expansions. The index 'N' denotes the order of a given NMP.

N	Symmetric nuclear matter		Symmetry energy	
0	ε_0	-16.0	J_0	32.0
1			L_0	50.0
2	K_0	230	$K_{\rm sym,0}$	-100
3	Q_0	-400	$Q_{\rm sym,0}$	550
4	Z_0	1500	$Z_{\rm sym,0}$	-750

Results and Discussions

The calculations are performed using two sets of priors (P1 and P2). For both the prior sets, e_0,K_0 and J_0 are taken to be Gaussian. For the set P1 remaining parameters are uniformly distributed but for prior set P2 most of the NMPs are assumed to have Gaussian distribution. These priors are used to construct the posterior distribution of the NMPs using the expansions E1 and E2 with their respective pseudo data for the NS matter EoS. The results are summarized in Table II : (i) the median values NMPs reconstructed from the pseudo data show larger deviations from their true values listed in Table I for second or higher order NMPs; (ii) the uncertainties on the NMPs are quite large (iii) the uncertainties on the Z_0 and $Z_{\rm sym,0}$ in Table II are considerably asymmetric about their median values. reflecting their non-

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Gaussian nature.

We have also examined the uncertainties in the NMPs that could arise from the allowed variations in the $\varepsilon(\rho, 0)$, $J(\rho)$ and δ for a given $\varepsilon(\rho, \delta)$.In short, for a given $\epsilon(\rho, \delta)$, the values of $J(\rho)$, $\varepsilon(\rho, 0)$ and δ may have some leeway. For the NMPs, we employ the marginalised PDs to obtain 68% and 95% confidence intervals for $\varepsilon(\rho, \delta)$, $\varepsilon(\rho, 0)$ and $J(\rho)$. The results are plotted only for the E2-P2 case in Fig. 1. At a specific density, the value of $\varepsilon(\rho, \delta)$ (top) varies within a small bound, but, $\varepsilon(\rho, 0)$ (middle) and $J(\rho)$ (bottom) have larger uncertainties. Due to the non-Gaussian nature of higher-order NMPs, Table II show that the 95% confidence intervals for $\varepsilon(\rho, 0)$ and $J(\rho)$ are slightly asymmetric compared to the ones for the 68%. Beyond $2\rho 0$, the dispersion in the values of $J(\rho)$ rises quickly with density. Spread in $J(\rho)$ at $4\rho_0$ is $\sim$ 36 MeV which increases to $\sim$ 160 MeV at $6\rho_0$, whereas, the spread in $\varepsilon(\rho, 0)$ remains almost the same ($\sim$ 15 MeV) for the density in the range $4\rho_0$ to $6\rho_0$, For 68% confidence interval.

TABLE II: The EoS for the neutron star matter is used to recover the posterior distributions for each nuclear matter parameter.

NMPs	E1-P1	E1-P2	E2-P1	E2-P2
ε_0	$-16.0^{+0.3}_{-0.3}$	$-16.0^{+0.3}_{-0.3}$	$-16.0^{+0.3}_{-0.3}$	$-16.0^{+0.3}_{-0.3}$
K_0	187^{+65}_{-56}	221^{+36}_{-28}	213^{+47}_{-40}	230^{+28}_{-25}
Q_0	-367^{+196}_{-220}	-471^{+113}_{-123}	-327^{+243}_{-198}	-412^{+159}_{-123}
Z_0	1518^{+258}_{-236}	1632^{+152}_{-157}	1307^{+1069}_{-1656}	1637^{+835}_{-1206}
J_0	$31.8^{+2.5}_{-2.6}$	$32.0^{+2.6}_{-2.7}$	$32.0^{+2.5}_{-2.5}$	$32.0^{+2.6}_{-2.4}$
L_0	52.8^{+25}_{-19}	55.5^{+17}_{-16}	$53.1^{+23.8}_{-19.3}$	$51.0^{+14.0}_{-13.9}$
$K_{\mathrm{sym},0}$	-34^{+142}_{-178}	-108^{+76}_{-72}	-114^{+113}_{-138}	-106^{+70}_{-70}
$Q_{\mathrm{sym},0}$	220^{+755}_{-563}	486^{+257}_{-264}	562^{+572}_{-488}	522^{+248}_{-241}
$Z_{\mathrm{sym},0}$	807^{+1341}_{-1527}	100^{+876}_{-668}	-60^{+1944}_{-1921}	-323^{+1920}_{-1643}

It should be noted that the nuclear matter parameter has a number of other sources of uncertainty. These sources of uncertainties include (i) inter-correlations of NMPs corresponding to $\varepsilon(\rho, 0)$ with the ones for $J(\rho)$, (ii) change in $J(\rho)$is compensated by appropriate change in asymmetry parameter δ and $\varepsilon(\rho, 0)$ in such a way that the EoS of neutron star matter remains more or less unaltered.

Thus it is not enough to reconstruct the NMPs from the EoS of NS matter only.The experimental data on the EoS of symmetric nuclear matter from the heavy-ion collision and the symmetry energy beyond the saturation density from the isobaric ana-log states may further help in constraining the nuclear matter parameters.

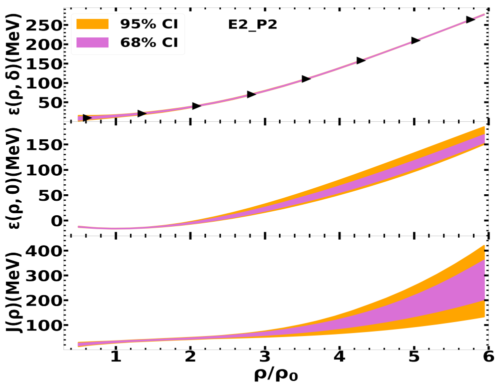

FIG. 1: Plots of 68% and 95% confidence intervals for the EoS for neutron star matter (top), the symmetric nuclear matter (middle), and the symmetry energy (bottom) as a function of scaled density with the prior set P2 for expansion E2 .

References

[1] Imam et al.,Phys. Rev. C,**105**, 015806 (2022).