Analysis of p-d elastic scattering cross section below the breakup threshold energy through PFM

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Introduction

Phase Function Method (PFM) is an efficient as well as a simplest method for dealing with quantum mechanical scattering problems involving both local and nonlocal interactions [1-3]. Unlike the local potential, in case of nonlocal potential the accumulation of the phase depends on the wave function for all values of r. Therefore, it is of considerable interest to apply PFM for nonlocal interaction. The modifications in traditional PFM for treating Coulomb/Hulthén distorted separable nonlocal potential have been reported in ref. [1,2]. The electromagnetic effect is to be quite important in low energy scattering demanding rigorous inclusion of the effect in the model. Generally, screened/cut off Coulomb potential is preferably used to represent the electromagnetic part of the charged hadronic systems as the poor asymptotic behavior of pure Coulomb one can affect the calculations of the theoretical results. In this context, the Manning-Rosen potential, satisfactorily serves as a model for the molecular as well as nuclear interaction [3,4]. Here we use Manning-Rosen potential as a short-range electromagnetic interaction adding to nonlocal separable potential in all partial waves for treating p-d elastic scattering thorough modified PFM.

Phase equation formalism

The radial Schrödinger equation satisfying the regular solution for Manning-Rosen plus Graz separable potential [1] is written as

$$\left\{\frac{d^2}{dr^2}+k^2-\frac{\ell(\ell+1)}{r^2}-V_M(r)\right\}\varphi_{MG}(k,r) = \lambda 2^{-2\ell}\left(\ell!\right)^{-2} r^{\ell} e^{-\beta r} d_\ell(k) \quad (1)$$

with $d_\ell(k)=\int_0^\infty dr'(r')^{\ell}\, e^{-\beta r'}\varphi_{MG}(k,r')$. (2)

For regular boundary condition, the solution $\varphi_{MG}(k,r)$ reads as

$$\phi_{MG}(k,r)=\phi_\ell^M(k,r)+\lambda\, 2^{-2\ell}\left(\ell!\right)^{-2} d_\ell(k)\int_0^r e^{-\beta r'}(r')^{\ell} G_\ell^{M(R)}(r,r')dr' . \quad (3)$$

Multiplying Eq. (3) with $r^{\ell}e^{-\beta r}$ on both sides and integrating from 0 to ∞ one can find

$$d_\ell(k)=\frac{1}{D_\ell(k)}\int_0^\infty e^{-\beta r}(r)^{\ell}\phi_\ell^M(k,r)\,dr , \quad (4)$$

where the Fredholm determinant associated with regular boundary condition

$$D_\ell(k)=1-\lambda 2^{-2\ell}\left(\ell!\right)^{-2} \times\int_0^\infty\int_0^r e^{-\beta r'}e^{-\beta r}(r')^{\ell}(r)^{\ell}G_\ell^{M(R)}(r,r')dr'dr . \quad (5)$$

The regular Green's function $G_\ell^{M(R)}(r,r')$ [1,2] can be written in modified form as

$$G_\ell^{M(R)}(r,r')=\frac{1}{\left|f_\ell^M(k)\right|^2}\Big[\phi_\ell^M(k,r) \times\mathrm{Re}\left\{f_\ell^M(k,r')f_\ell^M(-k)\right\}-\phi_\ell^M(k,r') \times\mathrm{Re}\left\{f_\ell^M(k,r)f_\ell^M(-k)\right\}\Big] \text{ for } r'<r$$
$$= 0 \text{ for } r'>r . \quad (6)$$

Substituting Eqs. (4) and (6) in Eq. (3), one can get

$$\phi_{MG}(k,r)=k\phi_\ell^M(k,r)\Bigg[1+\lambda\, 2^{-2\ell}\left(\ell!\right)^{-2} \times d_\ell(k)\frac{I_1}{k\left|f_\ell^M(k)\right|^2}\Bigg]-\lambda\, 2^{-2\ell}\left(\ell!\right)^{-2} \times d_\ell(k)\frac{I_2}{\left|f_\ell^M(k)\right|^2}\mathrm{Re}\left\{f_\ell^M(k,r)f_\ell^M(-k)\right\} \quad (7)$$

with

$$I_1 = \int_0^r (r')^\ell e^{-\beta r'} \,\mathrm{Re}\left\{ f_\ell^M(k,r') f_\ell^M(-k) \right\} dr' \qquad (8)$$

and $$I_2 = \int_0^r e^{-\beta r'} (r')^\ell \varphi_\ell^M(k,r') dr' \qquad (9)$$

where $\phi_\ell^M(k,r)$, $f_\ell^M(k,r)$ and $f_\ell^M(k)$ are regular solution, irregular solution and Jost function for the pure Manning-Rosen potential in all partial waves, respectively [4]. In terms of phase and amplitude functions, $\phi_{MG}(k,r)$ from equation (7) can be expressed as

$$\phi_{MG}(k,r) = \alpha_{MG}(k,r)\left[k\varphi_\ell^M(k,r)\cos\delta_{MG}(k,r) + \sin\delta_{MG}(k,r)\,\mathrm{Re}\left\{ f_\ell^M(k,r) f_\ell^M(-k) \right\} \right]. \qquad (10)$$

Here $\alpha_{MG}(k,r)$ and $\delta_{MG}(k,r)$ refers to the phase and amplitude function, respectively. Comparing Eq. (7) with (10) within the consideration of limiting condition $r \to \infty$, we have

$$\tan\delta_{MG}(k) = \frac{-\lambda\, 2^{-2\ell} (\ell!)^{-2} d_\ell(k) \dfrac{I_2(\beta,k)}{\left| f_\ell^M(k) \right|^2}}{1 + \lambda\, 2^{-2\ell} (\ell!)^{-2} d_\ell(k) \dfrac{I_1(\beta,k)}{k\left| f_\ell^M(k) \right|^2}} . \qquad (11)$$

Eq. (11) represents the desired phase equation involving the integrations $I_1(\beta,k)$ and $I_2(\beta,k)$ which in turn can be solved by exploiting basic properties and formulae for both homogenous and non-homogenous Gaussian hypergeometric functions [5, 6].

Results and discussion

The phase shifts for p-d scattering computed from our phase equation are compared with ref. [7]. Here $A/b = 0.0463\, fm^{-1}$.

Table 1: List of parameters for p-d system.

State	α	$\beta(fm^{-1})$	$\lambda(fm^{-2\ell-3})$	$b\ (fm)$
$^2S_{1/2}$	0.008	5.49	-276.8	7.92
$^2P_{1/2}$	0.099	1.38	-484.6	9.69
$^2P_{3/2}$	0.099	2.19	-1335.5	18.99

Table 2: Scattering phase shifts for p-d system.

E_{Lab}	$^2S_{1/2}$		$^2P_{1/2}$		$^2P_{3/2}$	
	PFM	Ref. [7]	PFM	Ref. [7]	PFM	Ref. [7]
1	-13.8	-14.1	-4.1	-2.9	-3.4	-2.6
2	-20.9	-20.4	-7.8	-7.6	-5.7	-5.7
3	-26.1	-24.1	-10.6	-11	-7.2	-7.2

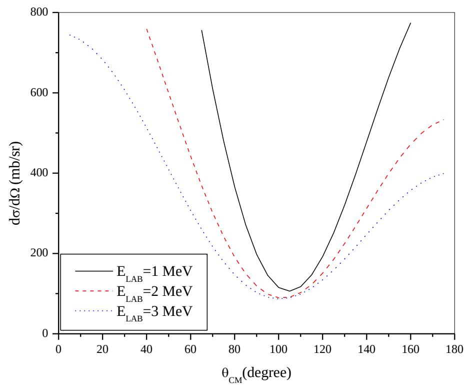

Figure 1: Differential cross-sections in center of mass system.

The phase parameters thus obtained in the energy range below the threshold for the deuteron break-up are, in turn, exploited to estimate the cross sections for the concerned system. Huttel et al [7] studied p-d elastic scattering phase shifts below the breakup threshold energies which are compared well with Faddeev calculations. In the present text we have used a model of five parameters central potential without including any spin-orbit and tensor interaction, that reproduces p-d data quite accurately. It is also worthwhile to mention that the overall quality of the consistency between our results and ref. [7] in the energy region under consideration, is notable.

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