Phase Shift Analysis of $\alpha - {}^{12}C$ Elastic Scattering Using Phase Function Method

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Introduction

Alpha-Carbon elastic scattering at low energies remains an intriguing subject and is regarded as the key process for determining the energy level of oxygen isotopes [1]. Shung-Ichi Ando [2] has approached the α-^{12}C elastic scattering at low energies by describing the two prticles as elementary-like fields and utilized effective field theory. Tischhauser et al. [3] reported elastic α-^{12}C phase analysis using R-matrix method. In recent past, Behera et al. [4] developed a precise analytical expression, for Fredholm determinant under motion in the Coulomb plus Graz separable potential, to investigate the phase shifts for the S, P and D partial waves at low energies. We have implemented a novel numerical solution [5] to solve the phase equation which deals with the potential directly. This procedure has been tested to successfully obtain the interaction potentials and resonances for various ℓ-channels of alpha-alpha scattering [6] and ground state of Triton(^3H) [7] and ^3He [8] systems. Here, we extend the procedure to obtain scattering phase shifts (SPS) in combination with optimization procedure to construct interaction potentials for P, D, F and G-partial wave states.

Formulation of the problem

The attractive nuclear potential for α-^{12}C system is modeled using Morse function as

$$V_{\rm M}(r) = V_0 \left(e^{-\frac{2(r-r_0)}{a_0}} - 2e^{-\frac{(r-r_0)}{a_0}} \right) \quad (1)$$

here the model parameters V_0, r_0 and a_0 describes strength, equilibrium distance and shape of Morse function respectively. The repulsive Coulomb potential is modeled using $erf()$-function as

$$V_{Coulomb}(r) = \frac{(Z_1 Z_2)e^2}{r} erf(\beta r) \quad (2)$$

here $Z_1 * Z_2 = 12$ and $\beta = \sqrt{3}/(2R_{\alpha C})$ with $R_{\alpha C} = 2.2478 fm$ root mean square radius of α-^{12}C. Thus total interaction potential is given by:

$$V_T(r) = V_{\rm M}(r) + V_{Coulomb}(r) \quad (3)$$

The scattering of α-particle (mass $M_\alpha = 3737.379$ MeV/c^2) with laboratory energy E_{lab} and orbital angular momentum ℓ scattering from Carbon of mass ($M_C = 11193.75$ MeV/c^2) can be transformed into center of mass reference frame. Then, the reduced mass, $\mu = M_\alpha * M_C/(M_\alpha + M_C)$, of two body system can be described using an interaction potential $V_T(r)$, as in Eq.3. The time independent Schrödinger equation (TISE) is given by

$$\frac{d^2 u_\ell(k,r)}{dr^2} + \left[k^2 - \frac{\ell(\ell+1)}{r^2} - U(r) \right] u_\ell(k,r) = 0) \quad (4)$$

where $U(r) = V_T(r)/(\hbar^2/2\mu)$ & $k_{c.m.} = \sqrt{E_{c.m.}/(\hbar^2/2\mu)}$. For $\alpha - {}^{12}C$ system, the value of $\hbar^2/2\mu = 6.96$ MeVfm^2 and centre of mass energy $E_{c.m.}$ is connected to laboratory energy $E_{\ell ab}$, by following relation, for non-relativistic kinematics:

$$E_{c.m.} = \left(\frac{m_C}{m_\alpha + m_C} \right) E_{lab} = 0.75 E_{lab} \quad (5)$$

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TABLE I: Optimized model parameters for all ℓ channels.

State	V_0 (MeV)	r_0 (fm)	a_0 (fm)
P	156.678	0.674	0.164
D	325.149	0.218	0.210
F	74.471	0.546	0.978
G	144.003	0.576	0.831

The phase equation, of Ricatti type first order non-linear differential equation, is given by

$$\delta'_\ell(k,r) = -\frac{U(r)}{k}\Big[\cos(\delta_\ell(k,r))\hat{j}_\ell(kr) - \sin(\delta_\ell(k,r))\hat{\eta}_\ell(kr)\Big]^2 \quad (6)$$

where $\hat{j}_\ell(kr)$ and $\hat{\eta}_\ell(kr)$ are the Riccati-Bessel and Riccati-Neumann functions of order ℓ.

Implementation of Numerical Solution

The above phase equations for various P, D, F and G-states are solved using fifth order Runge-Kutta method by choosing initial condition as $\delta'_\ell(k,0) = 0$ and integrating the RHS upto $r = 5fm$ in steps of 0.01, to obtain scattering phase shift (SPS).

Results and Discussion

The optimized model parameters for P, D, F and G-partial wave states, obtained by minimizing mean square error with respect to experimental SPS data taken from Plaga et al. [9], are shown in Table I. The SPS for various ℓ-channels are plotted in Fig.1**(top)** and the corresponding interaction potentials are shown in Fig.1**(bottom)**. Since all SPS are positive, the corresponding resultant potentials are seen to be having an attractive nature. It can be observed that the depths of these potentials does not show the ordering of the states as expected for $\alpha - {}^{12}C$ system. This is because they are consist of total potential V_T without addition of contribution due to centrifugal term. When one adds the later, one would clearly observe the correct ordering as well as scattering nature of these states.

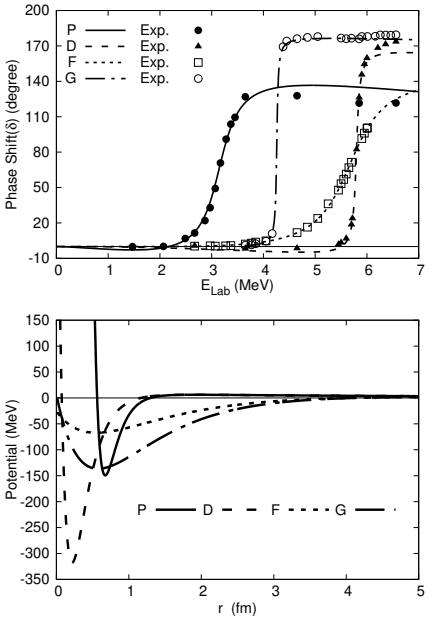

FIG. 1: Scattering phase shifts for P, D, F and G-channel (**top**), respective inverse potentials (**bottom**). The experimental scattering phase shifts are taken from Plaga et al. [9]

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