S-wave phase shifts for elastic nucleon-deuteron scattering using Malfliet-Tjon potential

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Introduction

Scattering experiments are one of the most important ways of probing to know the nature of interaction between various particles or system of particles, which in turn, would lead to knowledge of their inherent structure. Phase wave analysis of experimental scattering phase shifts (SPS) to obtain realistic potentials has been a standard approach followed for study of few bodys systems consisting of nucleon-nucleon, nulceon-nucleus and nucleus-nucleus interactions. In this paper, we look at study of SPS for nucleon-deuteron systems i.e. neutron-deuteron(nD) and proton-deuteron(pD) systems.

The three body nucleon-deuteron interaction has been studied remarkably by many groups, both theoretically and experimentally. However, phase shift study for these systems have been done only below three-body breakup threshold ([1] *and the references therein*). The extensive phase shift analysis for elastic nucleon-deuteron scattering above three-body breakup have been done by Huber *et.el.* [2] up-to 19 MeV and J.Arvieux [3] up-to 46.3 MeV for nD and pD interactions respectively and their data is widely accepted as a standard.

Our methodology to obtain SPS is using phase function method in tandem with optimisation techniques for determining model parameters [4] is akin to obtaining inverse potentials of interaction. Even though Morse potential is ideally suited as zeroth reference potential, one can in principle choose any other mathematical function for the purpose. Previosuly, we have studied nD interaction using Morse potential for ground state alone for energies upto 10.5 MeV. The model parameters were optimised by solving time independent Schrödinger equation to obtain the best value for binding energy of Triton [5]. Recently, we obtained SPS using Morse potential interaction for pD system for energies upto 10.04 MeV [6]. In this work, the very successful Malfliet-Tjon potential has been chosen to describe the nD and pD interactions, with an added Coulomb term for the later. The inverse potentials were determined by analysing the s-wave phase shifts (i.e for partial wave $\ell = 0$), for doublet ($^2S_{1/2}$) and quartet ($^4S_{3/2}$) states of nD and pD scattering.

Methodology

The Malfliet-Tjon (MT) [7] potential is given by

$$V(r) = V_{MT}(r) = -V_A\Big(\frac{e^{-\mu_A r}}{r}\Big)+V_R\Big(\frac{e^{-\mu_R r}}{r}\Big) \quad (1)$$

where, $\mu_R = 2\mu_A \ fm^{-1}$, thus making it a three parameter potential consisting of both attractive and repulsive parts.
For proton-deuteron (pD) interaction, Coulomb term is added given by [8],

$$V_C(r) = z_1 z_2 \frac{e^2}{r} erf(\beta r) \quad (2)$$

where $z_1 z_2 = 2$ for pD, $e^2 = 1.44 \ MeV fm$ and $\beta = \frac{\sqrt{3}}{2\times R_{pD}}$. Here, $R_{pD} = 1.9642 fm$ [6] is root mean square (rms) radius of proton-deuteron system, resulting in $\beta = 0.441 \ fm^{-1}$. The total interaction potential is utilised in phase function equation [4], given below, to obtain the scattering phase shifts.

$$\delta_0'(k,r) = -\frac{V(r)}{k} sin^2(kr + \delta_0(k,r)) \quad (3)$$

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Here, k is related to centre-of-mass energy by relation: $k_{cm} = \sqrt{\frac{E_{cm}}{\hbar^2/2\mu}}$; where

$$E_{cm} = \left(\frac{m_T}{m_P + m_T}\right) E_{lab} \tag{4}$$

Here, m_P and m_T are the masses of projectile and target respectively.

Results and Discussion

In order to obtain SPS, the non linear differential equation (NDE) Eq.(3) has been solved using Runge-Kutta (RK-5) method, from origin ($\delta(0) = 0$) to asymptotic region.
The model parameters have been optimised by minimising mean absolute percentage error (MAPE) as cost function for simulated phase shifts δ_i^o w.r.t. standard data δ_i^e [2, 3], as:

$$MAPE = \frac{1}{N} \sum_{i=1}^{N} \frac{|\delta_i^e - \delta_i^o|}{|\delta_i^e|} \times 100 \tag{5}$$

Potential parameters along with SPS have been tabulated in Table I and are ploted in Fig. 1 and 2. It is clear from Fig. 1, that the potential of $^2S_{1/2}$ state for both nD and pD scattering is deeper than that of respective $^4S_{3/2}$ states. Since our potentials are phase shift dependent, therefore the potential depth for both the states of pD scattering is of similar magnitude. Also the MAPE for obtained results with standard data [2, 3] is found to be less than 4% (Table I).

TABLE I: Model parameters of MT potential for $^2S_{1/2}$ and $^4S_{3/2}$ states of nD and pD scattering.

System	States	V_r	V_a	μ_A	$MAPE$
nD	$^2S_{1/2}$	13435.61	3012.23	0.80	1.93
nD	$^4S_{3/2}$	15.39	10.42	0.08	3.26
pD	$^2S_{1/2}$	1065.83	572.21	0.56	2.10
pD	$^4S_{3/2}$	316.39	218.85	0.79	3.51

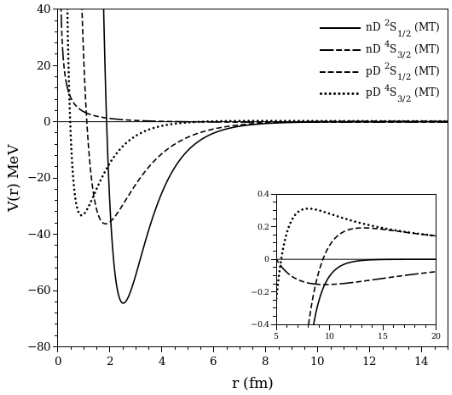

FIG. 1: nD and pD potentials for $^2S_{1/2}$ and $^4S_{3/2}$ states as a function of r. For pD system, Coulomb part of potential are clearly visible (inset)

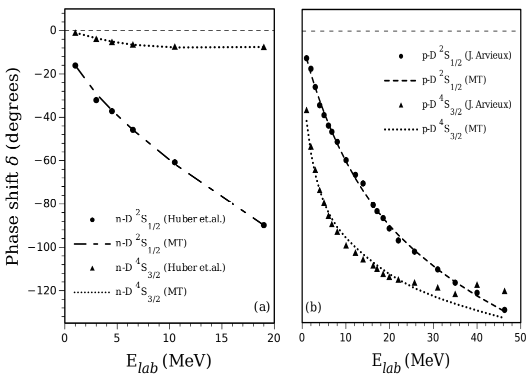

FIG. 2: SPS for (a) nD and (b) pD, $^2S_{1/2}$ and $^4S_{3/2}$ states as a function of laboratory energy.

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