Full implementation of post-form reaction theory in the electromagnetic breakup of exotic nuclei

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Introduction

Nuclei at the binding energy extremities can exhibit markedly exotic behaviour (such as developing a halo structure) compared to their stable isotopes. These weakly bound neutron-rich nuclei play a crucial role in the formation of heavier elements via radiative capture processes in suitable astrophysical environments. However, the existence of the Coulomb barrier leads to very low capture cross-sections at relevant Gamow energies, making it challenging to measure. Various indirect methods are used to calculate radiative capture cross-sections in this context. Coulomb dissociation (CD) is one of those alternative methods which relates the CD cross-section with photo-dissociation cross section, and subsequently invoking the principle of detailed balance, the radiative capture reaction rates at astrophysical energies can be calculated. Furthermore, breakup reactions have significant importance in the structural calculation of weakly bound nuclei.

In this contribution, we attempt to extend the existing theory of CD developed under the aegis of distorted wave Born approximation (DWBA) [1] without invoking any accuracy limiting approximations. To the best of our knowledge, this could be the first attempt to fully implement the post form transition amplitude in electromagnetic breakup. As an application, we focus on the Coulomb breakup of ^{11}Be impinging on a ^{208}Pb target at beam energy 72 MeV/u and discuss some preliminary results obtained from this full-order quantum mechanical theory.

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Formalism

The main objective is to solve the post form transition (T)-matrix which eventually results in the reaction cross sections. Consider a reaction $a + t \rightarrow b + c + t$, where the projectile '$a$' breaks up into fragments 'b' and 'c' in the Coulomb field of the target 't'. The position vector $\mathbf{r}$ of b with respect to t, and the projectile CM coordinate $\mathbf{r}_i$ with respect to t satisfy the relations: $\mathbf{r} = \mathbf{r}_i - \alpha\mathbf{r}_1$, $\mathbf{r}_c = \gamma\mathbf{r}_1 + \delta\mathbf{r}_i$; $\alpha = \frac{m_c}{m_c+m_b}$, $\delta = \frac{m_t}{m_b+m_t}$, $\gamma = (1-\alpha\delta)$, where $\mathbf{r}_1$ is the relative vector between b and c and m_i's ($i = a, b, c$) are masses of the concerned fragments. The exact post-form T-matrix is given by

$$T_{fi}^{(+)[post]} = \left\langle \chi_{q_b}^{(-)}(\mathbf{r})\chi_{q_c}^{(-)}(\mathbf{r}_c)\right| V_{bc}(\mathbf{r}_1) \left|\psi_i^{(+)}\right\rangle.$$

Here χ's are the full Coulomb distorted waves with ingoing boundary conditions for the relative motions of b and c with respect to t, respectively. $V_{bc}(\mathbf{r}_1)$ is the short range interaction potential between b and c in the initial channel. $\psi_i^{(+)}$ is the exact three body wave function of the system with outgoing boundary condition. In DWBA, the inelastic excitations of the projectile are considered to be small, and so the wave function is approximated by $\psi_i^{(+)} \approx \chi_{q_a}^{(+)}(\mathbf{r}_i)\Phi_a^{\ell m}(\mathbf{r}_1)$, where χ^+ and $\Phi_a^{\ell m}$ are the Coulomb distorted wave of the projectile and the bound state wave function of the projectile with orbital angular momentum content ℓ (projection m), respectively.

Finally, the post-form T-matrix can be written in terms of the reduced transition amplitude as,

$$\hat{\ell}\beta_{\ell m} = \int\int d\mathbf{r}_1 d\mathbf{r}_i e^{-\pi\eta_b/2}e^{-\pi\eta_c/2}e^{-\pi\eta_a/2}$$
$$\Gamma(1+i\eta_b)\Gamma(1+i\eta_c)\Gamma(1+i\eta_a)e^{-i\mathbf{q}_b\cdot\mathbf{r}}$$
$${}_1F_1(-i\eta_b, 1, i(q_b r + \mathbf{q}_b\cdot\mathbf{r}))e^{-i\mathbf{q}_c\cdot\mathbf{r}_c}$$

$$\times {}_1F_1(-i\eta_c, 1, i(q_c r_c + \mathbf{q}_c \cdot \mathbf{r}_c))V_{bc}(\mathbf{r}_1)$$
$$\times \Phi_a^{\ell m}(\mathbf{r}_1) e^{i\mathbf{q}_a \cdot \mathbf{r}_i} {}_1F_1(-i\eta_a, 1, i(q_a r_i - \mathbf{q}_a \cdot \mathbf{r}_i)).$$

where, η's are the Sommerfeld parameters of the respective fragments, $\mathbf{q}_i$'s $(i = a, b, c)$ are the respective Jacobi momenta, and $\hat{\ell} = \sqrt{2\ell + 1}$. ${}_1F_1$'s are confluent hypergeometric functions, which originate from the expressions of the Coulomb wave functions. The expression for $\beta_{\ell m}$ involves a six dimensional integration and is solved numerically. The triple differential cross-section can then be calculated using the following relation,

$$\frac{d^3\sigma}{dE_c d\Omega_b d\Omega_c} = \frac{2\pi}{\hbar \nu_{at}} \rho(E_c, \Omega_b, \Omega_c) \left(\frac{1}{2j_a + 1}\right) \times \sum_{\ell m} |\beta_{\ell m}|^2$$

where, ρ is the 3-body phase space factor, and j_a is the spin of the projectile.

Results and Discussions

As an application of our theory, we discuss the Coulomb breakup of ^{11}Be on a ^{208}Pb target at 72 MeV/u beam energy, i.e., ^{11}Be+^{208}Pb$\longrightarrow$ ^{10}Be+n+^{208}Pb. The ^{11}Be$-n$ bound state wave function is calculated using a Woods-Saxon potential by adjusting its depth to reproduce the one nuclei separation energy as $S_n = -0.504$ MeV.

A general code has been developed with Taylor series and asymptotic series methods to generate the confluent hypergeometric function [2] for a large range of arguments. The six dimensional integration has been evaluated numerically in the supercomputer PARAM Ganga, at IIT-Roorkee, using different Gauss quadrature methods. The Gauss-Legendre method is used to integrate over r_1 and the lab angles of the fragments and Gauss-Laguerre quadrature method is used for the r_i integration. As a preliminary result, we present the triple differential cross section for the emitted neutron in Fig. 1. The lab angle inputs are taken as $\theta_b = \theta_c = 2°$ and $\phi_b = \phi_c = 0°$. Interestingly, the peak of the distribution is at neutron energy 72 MeV which is consistent with the fact that the neutron is a "spectator" in this reaction.

Calculation of triple differential cross-section and hence other reaction observables, using this theory, are indeed computationally challenging. However, using the parallel programming tools like OpenMP and MPI, we can ameliorate this problem. Nevertheless, given that our method does not use any accuracy limiting approximation, other than the DWBA, we intend to present more accurate prediction of reaction observables in the breakup of exotic nuclei.

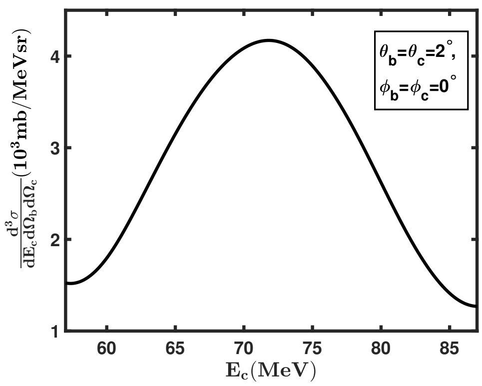

FIG. 1: Triple differential cross section for neutron energy distribution.

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