Application of Lagrange-mesh and R-matrix methods to nuclear transfer reactions involving resonances

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Introduction

Nuclear transfer reactions involving the transfer of particles (or a cluster) among the projectile and target, provide a useful tool to investigate the structure of a nucleus. Comparison of the measured cross sections with the calculated ones, can provides information like angular momentum (ℓ), spin-parity (J^π), spectroscopic factor (SF) or the asymptotic normalisation coefficient (ANC) of the populated state of a residual nucleus. In case, the final state is a resonance then the information about the width of resonance can be obtained by knowing the SF or the ANC of that particular state. These informations like, spin-parity, energy, widths etc. are often required in several estimation of astrophysical reaction rates and in those cases transfer reactions are used as an indirect tool for the study of such reactions.

There are different theoretical models for transfer reactions involving formalisms like, distorted wave approximation (DWBA), adiabatic method, continuum discretised coupled channel (CDCC) method and Faddeev method. Among all these, DWBA is the simplest approximation where the transfer is consider as one step process and since a long time it is being used for the analysis of experimental data. Modern calculations, like in the CDCC method are more demanding in terms of the computer capabilities and hence require efficient numerical techniques. In this regard, recently we have utilised the R-matrix method [1] along with the Lagrange-mesh method [2] in the DWBA formalism of transfer reactions [3–5]. These methods lead to the faster and accurate numerical computations. In this contribution we extend the application of our approach for the reactions involving resonances.

Formalism

We consider a transfer reaction $A(a+v)+t \rightarrow B(t+v)+a$, where a cluster v is transferred from the projectile A to the target t forming a residual nucleus B in the final state along with the the outgoing particle a.

The scattering matrix $U^{J\pi}_{\alpha\beta}$ for this stripping reaction is given by [3, 5]

$$U^{J\pi}_{\alpha\beta} = -\frac{i\sqrt{(SF_A)(SF_B)}}{\hbar} \times \left\langle \chi^{J\pi}_{L_B}(\boldsymbol{R}')\Phi^{I_B}_{\ell_B}(\boldsymbol{r}_B)|\mathcal{V}|\chi^{J\pi}_{L_A}(\boldsymbol{R})\Phi^{I_A}_{\ell_A}(\boldsymbol{r}_A)\right\rangle, \quad (1)$$

where χ_{L_A} and χ_{L_B} are the scattering wave functions in the initial and final channels for the relative motion of nuclei $A-t$ and $B-a$ with angular momenta L_A and L_B, respectively. $\Phi^{I_A}_{\ell_A}$ and $\Phi^{I_B}_{\ell_B}$ are the two-body bound state functions of nucleus A and B, having angular momenta ℓ_A and ℓ_B, respectively. I_k and SF_k are the total angular momentum and spectroscopic factor of nucleus k and labels α and β stand for (L_A, ℓ_A, I_A) and (L_B, ℓ_B, I_B). In the post-form DWBA, the interaction $\mathcal{V}$ is given by $\mathcal{V} = V_{av} + U_{at} - U_{aB}$, where V_{ij} is the binding potential and U_{ij} are the scattering potential between $i-j$. The latter two terms are called remnant interactions and they often appear similar.

Using the R-matrix method one can divide the scattering wave function χ into two parts, the internal and the external, at some channel radius a which is considered as large enough so that nuclear interactions become negligible.

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In the external region ($R > a$), the wave function take the asymptotic form

$$\chi_L^{ext}(R) = \sqrt{\frac{1}{v}}[I_L(kR) - U_L O_L(kR)], \quad (2)$$

where I_L and O_L are the incoming and outgoing Coulomb functions, k is the wave number and U_L is the scattering matrix for the elastic scattering. On the other hand, in the internal region we expand the wave function over a set of N Lagrange basis functions φ_i(R) as

$$\chi_L^{int}(R) = \sum_{i=1}^{N} c_i^L \varphi_i(R), \quad (3)$$

where c_i are the expansion coefficients and can be easily determined along with the elastic scattering matrix U_L [1, 3]. Normally, one needs $N \approx 30-40$ mesh points for a good convergence which are significantly lesser than the number of points needed in other methods like finite-difference method (typically of the order of 500). For more details of the formalism one is referred to Refs. [3, 5].

Results and discussions

We consider the reaction ^{6}He$(d,p)^7$He(g.s.), where the final nucleus state is a resonance state ($J^\pi = 3/2^-$) with width $\Gamma = 150 \pm 20$ keV and lies at around 410 keV (*nndc.bnl.gov*).

The bound state wave function $\Phi_{\ell_B}^{I_B}$ in this case is replaced by the bin function which is a superposition of scattering states over the width of the resonance. We use global parametrization of Ref. [6] for the nucleon optical potentials and the deuteron potential is obtained from the optical potentials of proton and neutron in the Watanabe form.

In Fig. 1, we plot the angular distribution of the α-transfer reaction ^{6}He$(d,p)^7$He(g.s.) at $E_{c.m.} = 17.25$ MeV and compare it with the data from Ref. [7]. One can see that the shape of the experimental data is reproduced nicely. In the above calculations we have used a SF of 1 as our goal is not to fit the data. Our aim is to show that with the current approach one can deal with the transfer processes involving resonances.

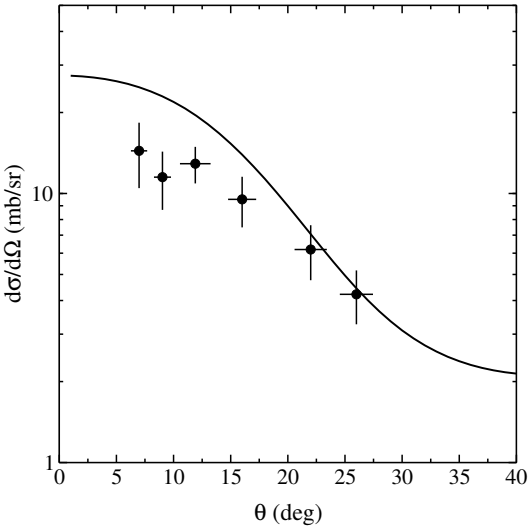

FIG. 1: Angular distribution of the cross section for the ^{6}He$(d,p)^7$He(g.s.) at $E_{c.m.} = 17.25$ MeV. Experimental data are taken from Ref. [7].

We will present the details of our calculations and will discuss some other cases of nucleon and cluster transfer.

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