Sub-barrier fusion hindrance and absence of neutron-transfer channels

Vinay Singh,* Joydev Lahiri, and D. N. Basu
Variable Energy Cyclotron Centre, 1/AF Bidhan Nagar, Kolkata 700064

Introduction

The sub-barrier fusion hindrance has been observed in the domain of very low energies of astrophysical relevance. This phenomenon can be analyzed effectively using an uncomplicated straightforward elegant mathematical formula derived presuming diffused barrier with a Gaussian distribution. The mathematical formula for cross section of nuclear fusion reaction has been obtained by folding together a Gaussian function representing the fusion barrier height distribution and the expression for classical cross section of fusion assuming a fixed barrier. The variation of fusion cross section as a function of energy, thus obtained, describes well the existing data on sub-barrier heavy-ion fusion for lighter systems of astrophysical interest. Employing this elegant formula, cross sections of interacting nuclei from ^{16}O + ^{18}O to ^{12}C + ^{198}Pt, all of which were measured down to < 10 μb have been analyzed. The agreement of the present analysis with the measured values is comparable, if not better, than those calculated from more sophisticated calculations. The three parameters of this formula varies rather smoothly implying its usage in estimating the excitation function or extrapolating cross sections for pairs of interacting nuclei which are yet to be measured. Possible effects of neutron transfers on the hindrance in heavy-ion fusion have been explored.

Theoretical formalism

In order to replicate the dependence of nuclear cross sections of fusion reaction on the collisional kinetic energy, specifically measured at low energies near fusion threshold, the assumption of a fusion barrier height distribution becomes necessary to simulate the effects resulting from the coupling to other channels. In the coupled-channel calculations it is naturally achieved which involve, in both colliding nuclei, the coupling to collective states down to the lowest level. The nuclear structure effects influencing the distribution of potential energy barriers have been considered negligible and hence ignored in this work. A Gaussian form $D(h)$ simulating the shape of the diffused barrier has been conceptualized [1] for the fusion barrier height distribution. Therefore, the distribution of barriers is provided by $D(h) = \frac{1}{\sqrt{2\pi}\sigma_h} \exp\Big[-\frac{(h-h_0)^2}{2\sigma_h^2}\Big]$ where for each individual reaction, the parameters h_0 (mean barrier height) and σ_h (width of barrier distribution) are to be determined exclusively. A mathematical formula for the cross section can be derived for surmounting the barrier arising due to interacting nuclei by folding the Gaussian distribution for fusion barriers with classical nuclear fusion reaction cross section expression $\sigma_f(h)=\pi R_h^2\Big[1-\frac{h}{E}\Big]\ \forall\ h \le E$, and $0\ \forall\ h \ge E$ where R_h marks, approximately, the position of the barrier (effective radius corresponding to the relative distance), which results in the following expression

$$\begin{aligned}\sigma_c(E) &= \int_{E_0}^{\infty} \sigma_f(h)D(h)dh \\ &= \int_{E_0}^{h_0} \sigma_f(h)D(h)dh + \int_{h_0}^{E} \sigma_f(h)D(h)dh \qquad (1)\\ &= \pi R_h^2 \frac{\sigma_h}{E\sqrt{2\pi}}\Big[\xi\sqrt{\pi}\Big\{\mathrm{erf}(\xi)+\mathrm{erf}(\xi_0)\Big\} + e^{-\xi^2} - e^{-\xi_0^2}\Big]\end{aligned}$$

where $E_0 = 0$ for positive Q value reactions and $E_0 = Q$ for negative Q value reactions, Q value being the sum of the rest masses of fusing nuclei minus rest mass of the resultant

*Electronic address: vsingh@vecc.gov.in

fused nucleus, $\xi = \frac{E-h_0}{\sigma_h\sqrt{2}}$, $\xi_0 = \frac{h_0-E_0}{\sigma_h\sqrt{2}}$ and erf(ξ) is the Gaussian error integral for argument ξ. The three parameters R_h, h_0 and σ_h have to be determined by a least square fitting of Eq.(1) to the measured excitation functions for fusion reactions. While deriving the formula of Eq.(1), the quantal effects of barrier penetration have not been taken explicitly into account. The structure of a given excitation function for fusion reaction is, however, influenced by the sub-barrier tunneling which has been included effectively through the parameter σ_h which describes the width of barrier distribution.

Calculations and Results

In order to envision the conditions of overcoming the potential barrier in nuclear collisions and to have a systematic knowledge on the essential characteristics, *viz.* h_0 (mean barrier height), σ_h (width) and R_h (effective radius), of the interacting potential, a set of accurately measured excitation functions of fusion reactions has been studied for two colliding nuclei. The values of the essential parameters h_0, σ_h and R_h are determined using the method of least-square fit.

TABLE I: The extracted values of h_0, σ_h and R_h, obtained from the analyses of the measured fusion excitation functions. Z_i and A_i are the charge and mass numbers of interacting nuclei, respectively, $A_{12} = A_1^{1/3} + A_2^{1/3}$ and $z = Z_1Z_2/A_{12}$. The table is arranged in order of increasing projectile mass.

Reaction	z	A_{12}	σ_h	h_0	R_h
[Expt.Ref.]			[MeV]	[MeV]	[fm]
$^{12}C+^{24}Mg$ [2]	13.916	5.174	0.815	11.483	6.646
$^{12}C+^{30}Si$ [3]	15.565	5.397	1.090	13.540	8.300
$^{12}C+^{198}Pt$ [4]	57.650	8.118	1.749	55.140	11.179
$^{16}O+^{18}O$ [2]	12.450	5.141	0.859	9.797	7.743
$^{28}Si+^{64}Ni$ [5]	55.709	7.037	1.402	50.403	7.182
$^{58}Ni+^{58}Ni$ [6]	101.269	7.742	2.275	98.278	8.550
$^{64}Ni+^{64}Ni$ [6]	98.000	8.000	1.466	92.646	8.862

The one neutron and two neutron transfer Q-values from projectile to target nuclei Q_{1n} and Q_{2n}, respectively, and those from target to projectile nuclei (except Q_{2n} of transfer from target to projectile nuclei for $^{28}Si+^{64}Ni$) are all negative. The reactions studied in the present work, the transfer effects are, therefore, absent leading to fusion hindrance. Hence the width σ_h of the distribution function is not enhanced due to any mixture of transfer and deformation effects.

Summary and Conclusion

In the region of sub-barrier energies, the fusion reaction cross sections have been estimated spanning a broad energy range. A Gaussian distribution function for the barrier heights is assumed to derive a simple diffused-barrier formula. The values of the essential parameters h_0 (mean barrier height), σ_h (width) and R_h (effective radius) are determined by least-square fitting Eq.(1) to the measured excitation functions. In the fusion reactions studied here, the transfer effects are absent leading to fusion hindrance. The widths of the barrier distribution are not enhanced due to any mixture of transfer and deformation effects. Even for positive fusion Q-values, the fusion hindrance remains because of small σ_h. This effect may be attributed to the saturation properties of nuclear matter, which prevents substantial overlap of light nuclei participating in the fusion reactions causing hindrance in quantum tunneling.

Acknowledgments

One of the authors (DNB) acknowledges support from Science and Engineering Research Board, Department of Science and Technology, Government of India, through Grant No. CRG/2021/007333.

References

[1] K. Siwek-Wilczyńska and J. Wilczyński, Phys. Rev. **C 69**, 024611 (2004).
[2] G. Montagnoli et al., Phys. Rev. **C 101**, 044608 (2020).
[3] G. Montagnoli et al., Phys. Rev. **C 97**, 024610 (2018).
[4] A. Shrivastava et al., Phys.Lett. **B 755**, 332 (2016).
[5] P. W. Wen et al., Phys. Rev. **C 103**, 054601 (2021).
[6] M. Rashdan, Phys. Rev. **C 102**, 055804 (2020).