Revisiting the Fractional Moment of Inertia of Rotating Neutron Stars

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Introduction

A neutron star (NS) is a remnant of a massive star that ran out of fuel and died in a supernova explosion. These stars, observed as pulsars, have very stable periods. However, it is observed that the spin frequency of a neutron star sometimes shows a sudden increase, exhibiting a pulsar glitch. An important parameter related to glitches is the fractional moment of inertia (FMI), defined as the ratio of crystal moment of inertia (MOI) to the overall MOI of the NS. In this work, we present the FMIs for rotating NSs and try to explain glitch observations using various equations of states (EOSs).

Formalism

The MOI for a spheroidal NS with rotational frequency ν is

$$I = \frac{2}{\nu}\int_0^{\pi/2} d\theta \int_0^{R(\theta)} dr P^2(r,\theta) \\ Q^2(r,\theta)[\epsilon(r,\theta)+p(r,\theta)]\frac{W(r,\theta)}{1-W^2(r,\theta)}r^3 sin^2\theta, \tag{1}$$

and the crustal MOI can be written as [1]

$$\Delta I = \frac{2}{\nu}\int_0^{\pi/2} d\theta \int_{R_t(\theta)}^{R(\theta)} dr P^2(r,\theta) \\ Q^2(r,\theta)[\epsilon(r,\theta)+p(r,\theta)]\frac{W(r,\theta)}{1-W^2(r,\theta)}r^3 sin^2\theta. \tag{2}$$

where the symbols have their usual meaning. The FMI is defined as the ratio of $\Delta I/I$.

The FMI can be evaluated from pulsar glitch observations using

$$\Delta I/I > 2\tau_c \frac{1}{t_i}\left(\frac{\Delta\nu}{\nu}\right)_i. \tag{3}$$

Here $\frac{\Delta\nu}{\nu}$ is the fractional rise in spin frequency, t_i is the time preceding the last glitch, and τ_c is the characteristic age of the pulsars.

Results and Discussions

To study the FMI of rotating NSs, we have considered two unified EOSs, IOPB and BKA24 [2].

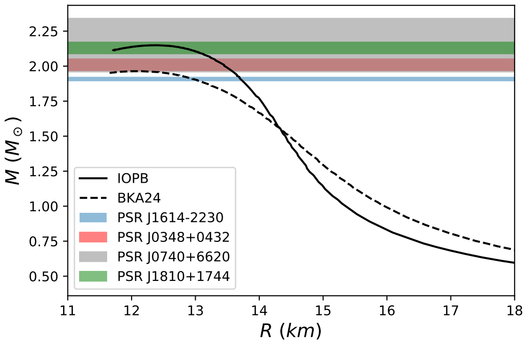

FIG. 1: The mass-radius curve for the IOPB and BKA24 EOSs. The shaded region represents constraints obtained from astronomical observations.

The mass-radius curve for these EOSs is shown in Fig. 1, where we have employed a few mass constraints from observations [1]. The plot shows that BKA24 satisfies a few constraints, whereas IOPB satisfies all four plotted constraints.

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We have explored the variation of FMI for NSs rotating with different rotational frequencies as a function of mass and radius, by suitably adapting the `LORENE` [3] libraries. The FMI shows a slight variation with mass (Fig. 2), whereas it shows a significant variation with radius (Fig. 3) for different rotational frequencies.

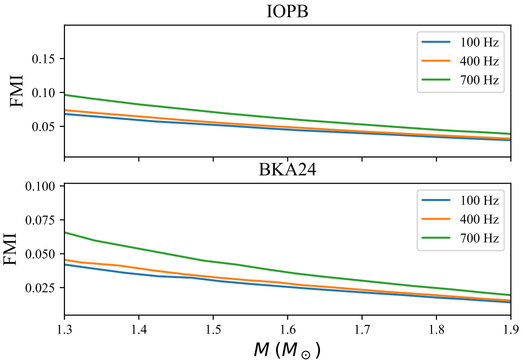

FIG. 2: The FMI of NSs with different rotational frequencies as a function of mass for both IOPB and BKA24 EOSs.

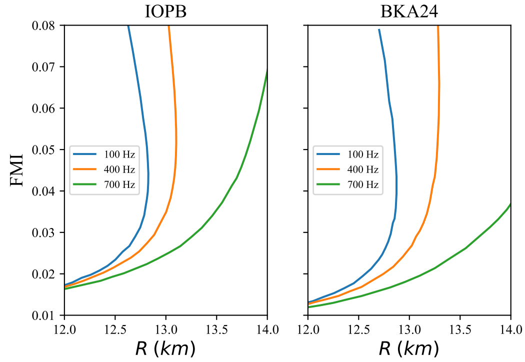

FIG. 3: The FMI of NSs with different rotational frequencies as a function of radius for both IOPB and BKA24 EOSs.

The distribution of FMI for 173 large glitches ($\Delta\nu/\nu > 10^{-6}$) from the Jodrell Bank glitch catalogue [4] is shown in Fig. 4. The dashed lines represent FMI for various EOSs. IOPB cannot explain 26.27%, whereas BKA24 failed to explain 34.75% of large glitches.

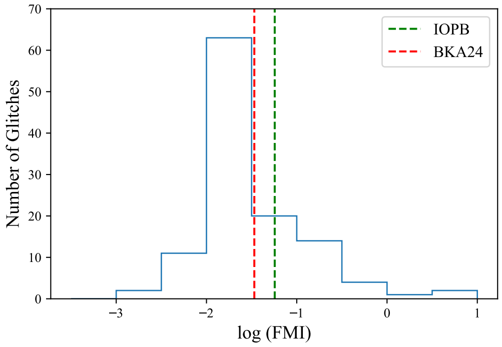

FIG. 4: The distribution of FMIs calculated for large glitches from the glitch catalogue in the Jodrell Bank Observatory. The dashed lines represent FMI for various EOSs.

Conclusion

The EOSs failed to explain a significant fraction of the observed large glitches. As suggested in various works [1], this could be because the crustal superfluid is not sufficient to account for the momentum transfer during a glitch. Thus, it is necessary to invoke superfluidity in the NS cores through some mechanism. This may be possible by implementing superfluid dynamics in the NS core in the framework of two fluid formalism, and accounting the occurrence of glitches to the transfer of MOI between these fluids.

HG, JS and AK acknowledges the financial support from MoE (Govt. of India) for their Ph.D. thesis work.

References

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